Book 1

Year 11

Mathematics

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Year 11 Book One

GOVERNMENT OF SAMOA
DEPARTMENT OF EDUCATION

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Section 1.1 Working With Real Numbers

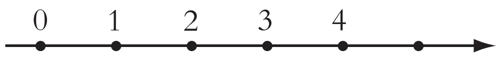
Recognising Real Numbers

Numbers can be put into groups.

1. **Natural Numbers (N)**
   Natural numbers are the counting numbers.

   N = {1, 2, 3, 4 . . .}

2. **Whole Numbers (W)**
   Whole numbers are the natural numbers and 0.



   W = {0, 1, 2, 3, 4 . . .}

3. **Integers (I)**
   Integers are the positive and negative counting numbers and 0.

   I = {. . . ⁻4, ⁻3, ⁻2, ⁻1, 0 , 1, 2, 3, 4 . . .}

4. **Rational Numbers (Q)**
   Rational numbers can be written as a fraction.

   $4 = \frac{4}{1}$ $\qquad -3 = \frac{^{-}12}{4}$ $\qquad 1.5 = \frac{3}{2}$

5. **Irrational Numbers**
   Irrational numbers cannot be written as a fraction.

   For example: π (we use an approximation of $\frac{22}{7}$)

   $\sqrt{2}$

6. **Terminating Decimals**
   These are decimals numbers which stop after a certain number of decimal places.

   For example: $\frac{7}{8} = 0.875$

   This is a terminating decimal because it stops (terminates) after 3 decimal places.

7. **Recurring Decimals**
   These are decimals numbers which keep repeating a digit or group of digits.
   For example: $\frac{137}{259} = 0.528\ 957\ 528\ 957\ 528\ 957\ \ldots$

   This is a recurring decimal. The six digits 528957 repeat in this order. Recurring decimals are written with dots over the first and last digit of the repeating digits: e.g. $0.\dot{5}28\ 95\dot{7}$

Note: All terminating and recurring decimals can be written in the form $\frac{m}{n}$. Therefore, they are rational numbers.

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8. **Real Numbers**
   These are made up of all possible rational and irrational numbers.

Example 1
Classify the following numbers as integers, rational, irrational, recurring decimals, terminating decimals.

$\frac{5}{7}$ $\quad -7 \quad 0.6 \quad 0.\dot{4}121\dot{3} \quad \frac{5}{8} \quad 11 \quad \sqrt{10} \quad \frac{\pi}{4} \quad \sqrt{49}$

Solution

Number	**Rational**	**Irrational Decimal**	**Integer Decimal**	**Recurring**	**Terminating**
$\frac{5}{7}$	✓			$0.\dot{7}1428\dot{5}$	
−7	✓		✓		
0.6	✓				✓
$0.\dot{4}121\dot{3}$	✓			✓	
$\frac{5}{8}$	✓				0.625
11	✓		✓		
$\sqrt{10}$		✓			
$\frac{\pi}{4}$		✓			
$\sqrt{49}$	✓		✓		

Example 2
Show that the number 0.345 is rational.

Solution

$$0.345 = \frac{345}{1000} = \frac{69}{200}$$

It can be written as a fraction, therefore it is rational.

Skill Exercises: Recognising Real Numbers

1. Classify the following numbers as rational, irrational, terminating or recurring decimals.

   (a) $\sqrt{100}$ (b) 0.6 (c) $\frac{\pi}{0.2}$ (d) $\frac{13}{99}$

   (e) 0.75 (f) $\frac{1}{\pi}$ (g) $\sqrt{11}$ (h) $\frac{1.6}{4}$

   (i) π^2 (j) $\frac{5}{11}$

2. Where possible write each of the following numbers in the form $\frac{m}{n}$, where m and n are integers with no common factors.

   (a) 0.49 (b) $0.\dot{3}$ (c) $\frac{\sqrt{49}}{4}$ (d) $\sqrt{7}$

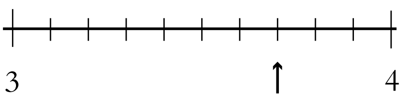
   (e) 0.417 (f) 0.1 (g) $0.\dot{0}\dot{9}$ (h) $\frac{\sqrt{36}}{\sqrt{121}}$

   (i) 0.125 (j) 0.962

Graphing Real Numbers and Integers on Number Lines

A number line is a straight line. It is possible to mark all the real numbers on this line.

Example 1
What number is the arrow pointing to?

(a)


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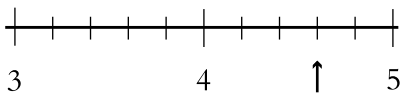

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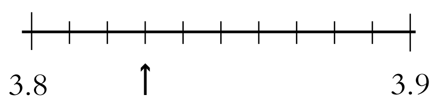
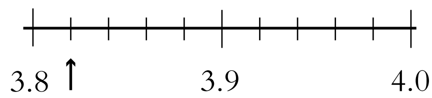


(d)


Solution

(a) Each mark on the scale is 0.1 units apart, so the arrow points to 3.7.

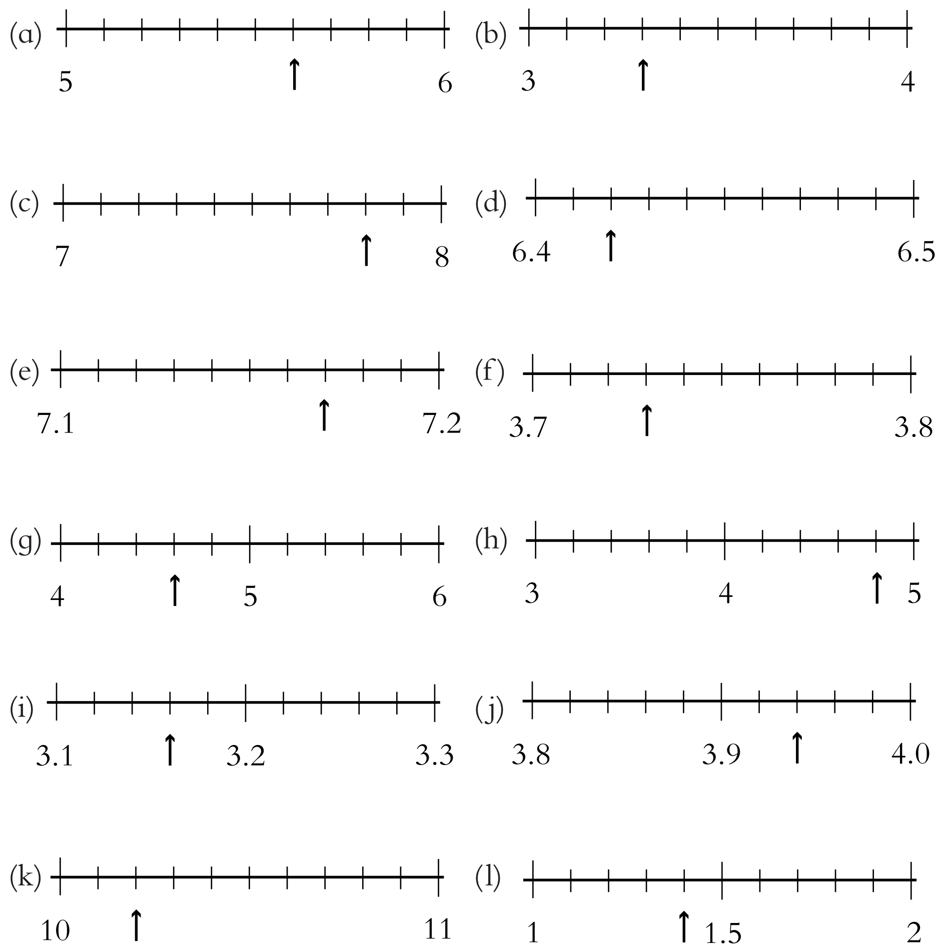
(b) Each mark on the scale is 0.2 units apart, so the arrow points to 4.6.

(c) Each mark on the scale is 0.01 units apart, so the arrow points to 3.83.

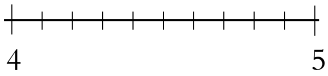
(d) Each mark on the scale is 0.02 units apart, so the arrow points to 3.82.

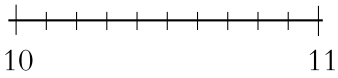
Skill Exercises: Graphing Real Numbers

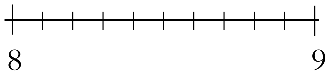
1. What number is the arrow pointing to?

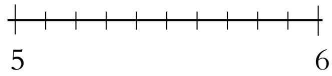


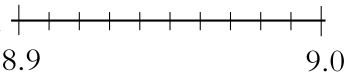
2. On a copy of the scale, mark as accurately as possible the given number.

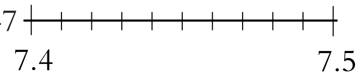
(a) 4.6 

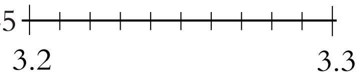
(b) 10.4 

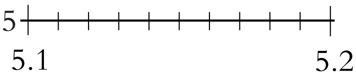
(c) 8.7 

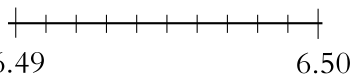
(d) 5.45 

(e) 8.91 

(f) 7.47 

(g) 3.245 

(h) 5.175 

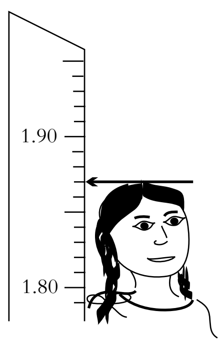
(i) 6.495 

3. In each number below, does the 5 represent: 5 tenths, 5 hundredths or 5 thousandths?

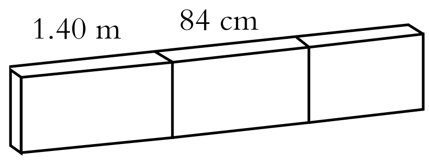
(a) 0.152 (b) 0.522 (c) 0.05

(d) 3.572 (e) 1.475 (f) 3.115

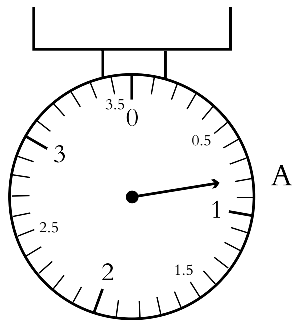
4. Makareta measures her height. How tall is she?



5. Perenise marks a 3 metre length of wood into three pieces. One piece is 1.40 metres long. Another piece is 84 centimetres long. How long is the third piece of wood?



6. These scales weigh in kilograms. Write down the weight when the pointer is at A.



Section 1.2 Working With Negative Exponents

Revising Index Notation

Index notation is a useful way of writing expressions like:

$2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

in a shorter format. This could be written with index notation as 2^7. The smaller number, 7, is called the index or power.

Example 1

Find:

(a) 3^4 (b) 4^5 (c) 7^1

Solution

(a) $3^4 = 3 \times 3 \times 3 \times 3 = 81$

(b) $4^5 = 4 \times 4 \times 4 \times 4 \times 4 = 1024$

(c) $7^1 = 7$

Example 2

Find the missing number:

(a) $3^4 \times 3^6 = 3^?$ (b) $4^2 \times 4^3 = 4^?$ (c) $\dfrac{5^7}{5^4} = 5^?$

Solution

(a) $3^4 \times 3^6 = (3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3 \times 3)$
$= 3^{10}$

(b) $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4)$
$= 4^5$

(c) $$\frac{5^7}{5^4} = \frac{5 \times 5 \times 5 \times \cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1} \times \cancel{5}^{1}}{\cancel{5}_{1} \times \cancel{5}_{1} \times \cancel{5}_{1} \times \cancel{5}_{1}}$$
$= 5 \times 5 \times 5$
$= 5^3$

Note: $a^m \times a^n = a^{m+n}$ and $\dfrac{a^n}{a^m} = a^{n-m}$

These rules apply whenever index notation is used. Using these rules:

$$\frac{a^3}{a^3} = a^{3-3} = a^0 \quad \text{or} \quad \frac{a^3}{a^3} = \frac{\cancel{a}^{1} \times \cancel{a}^{1} \times \cancel{a}^{1}}{\cancel{a}_{1} \times \cancel{a}_{1} \times \cancel{a}_{1}} = 1$$

$\therefore\ a^0 = 1$

Example 3

Find:

(a) $(2^3)^4$ (b) $(3^2)^3$

Solution

(a) $(2^3)^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 2 \times 2)$
$= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
$= 2^{12}$

(b) $(3^2)^3 = (3 \times 3) \times (3 \times 3) \times (3 \times 3)$
$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 3^6$

Note: $(a^m)^n = a^{m \times n}$

Skill Exercises: Revising Index Notation

1. Write each of following using index notation:

(a) $4 \times 4 \times 4 \times 4 \times 4$ (b) $3 \times 3 \times 3$

(c) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$ (d) $7 \times 7 \times 7 \times 7$

(e) $18 \times 18 \times 18 \times 18$ (f) 19×19

(g) $4 \times 4 \times 4 \times 4 \times 4 \times 4$ (h) $7 \times 7 \times 7 \times 7 \times 7$

(i) $10 \times 10 \times 10 \times 10 \times 10 \times 10$ (j) $100 \times 100 \times 100 \times 100 \times 100$

2. Find the value of each of the following:

(a) 3^4 (b) 5^4 (c) 7^4 (d) 10^4

(e) 5^0 (f) 3^6 (g) 2^7 (h) 2^1

(i) 8^4 (j) 4^1 (k) 3^0 (l) 5^2

3. Fill in the missing numbers:

(a) $2^7 \times 2^4 = 2^?$ (b) $3^4 \times 3^5 = 3^?$ (c) $3^6 \times 3^7 = 3^?$

(d) $4^? \times 4^2 = 4^7$ (e) $5^? \times 5^2 = 5^6$ (f) $5^4 \times 5^? = 5^9$

(g) $?^2 \times 4^4 = 4^6$ (h) $5^7 \div 5^4 = 5^?$ (i) $3^4 \div 3^2 = 3^?$

(j) $7^{14} \times 7^{10} = 7^?$ (k) $17^5 \div 17^? = 17^3$ (l) $9^7 \div 9^? = 9^3$

(m) $4^6 \times 4^? = 4^{11}$ (n) $4^? \div 4^6 = 4^{10}$ (o) $3^? \times 3^2 = 3^8$

(p) $3^6 \div 3^6 = ?$ (q) $3^7 \div 3^6 = ?$ (r) $3^0 \times 3^? = 3^5$

(s) $3^0 \times 3^7 = 3^?$ (t) $4^1 \times 4^? = 4^8$ (u) $5^2 \times 5^? = 5^2$

4. Fill in the missing numbers:

(a) $4 = 2^?$ (b) $8 = 2^?$ (c) $16 = 2^?$

(d) $64 = 2^?$ (e) $27 = 3^?$ (f) $25 = 5^?$

(g) $64 = 4^?$ (h) $81 = 3^?$ (i) $125 = ?^3$

5. Simplify the following expressions giving your answer in index notation:

(a) $3^7 \times 3^6 =$ (b) $2 \times 2^7 =$ (c) $4^5 \times 4^6 =$

(d) $3^6 \times 3^4 =$ (e) $2^4 \times 2^5 =$ (f) $2^6 \times 2^4 =$

(g) $3^7 \div 3^2 =$ (h) $3 \times 3^6 =$ (i) $3^6 \div 3 =$

(j) $\dfrac{8^{12}}{8^2} =$ (k) $\dfrac{7^6}{7^3} =$ (l) $\dfrac{9^2}{9^0} =$

(m) $4 \times 2^2 =$ (n) $\dfrac{2^5}{4} =$ (o) $\dfrac{2^6}{8} =$

6. Fill in the missing powers:

(a) $8 = 2^?$ (b) $1000 = 10^?$ (c) $16 = 2^?$

(d) $27 = 3^?$ (e) $81 = 3^?$ (f) $10\,000 = 10^?$

(g) $625 = 5^?$ (h) $64 = 4^?$ (i) $1296 = 6^?$

(j) $1 = 2^?$ (k) $36 = 6^?$ (l) $1 = 5^?$

7. Simplify the following giving your answers in index form:

(a) $(2^3)^2 =$ (b) $(3^2)^2 =$ (c) $(6^2)^3 =$

(d) $(5^3)^2 =$ (e) $(2^2)^4 =$ (f) $(4^2)^3 =$

(g) $(3^2)^4 =$ (h) $(5^2)^4 =$ (i) $(3^3)^2 =$

8. Fill in the missing numbers:

(a) $(2^2)^4 = 2^?$ (b) $(2^?)^3 = 2^{12}$ (c) $(3^2)^5 = ?^{10}$

(d) $(5^?)^4 = 5^{12}$ (e) $(10^5)^? = 10^{15}$ (f) $(7^5)^? = 7^{20}$

9. Simplify each of the following giving your answer in index notation:

(a) $3^2 \times 3^0 \times 3^4 =$ (b) $2^6 \times 2^7 \times 2 =$ (c) $5^2 \times 5^7 \times 5^3 =$

(d) $\dfrac{7^2 \times 7^4}{7^3} =$ (e) $\dfrac{7^4 \times 7^5}{7^2 \times 7^3} =$ (f) $\dfrac{2^3 \times 2^8}{2^3 \times 2} =$

(g) $\dfrac{3^2 \times 3^3}{3^5} =$ (h) $\dfrac{4^7 \times 4^8}{4^5 \times 4^9} =$ (i) $\dfrac{2^3 \times 2^0}{2^2} =$

10. Simplify each of the following expressions:

(a) $a^3 \times a^2 =$ (b) $a^4 \times a^6 =$ (c) $x^2 \times x^7 =$

(d) $x^4 \div x^2 =$ (e) $y^3 \times y^0 =$ (f) $p^7 \div p^4 =$

(g) $q^6 \div q^3 =$ (h) $x^7 \times x =$ (i) $b^4 \div b =$

(j) $\dfrac{b^6}{b^0} =$ (k) $\dfrac{c^7}{c^4} =$ (l) $\dfrac{x^8}{x^3} =$

(m) $\dfrac{y^3}{y} =$ (n) $\dfrac{x^4}{x^4} =$ (o) $x^2 \times x^3 \times x^3 =$

(p) $\dfrac{p^2 \times p^7}{p^5} =$ (q) $\dfrac{x^{10}}{x^2 \times x^5} =$ (r) $\dfrac{y^3 \times y^7}{y^2 \times y^4} =$

(s) $\dfrac{x^2 \times x^3}{x^5} =$ (t) $\dfrac{x^7 \times x}{x^3 \times x^4} =$ (u) $\dfrac{x^8 \times x^4}{x^0} =$

(v) $(x^2)^4 =$ (w) $(x^3)^5 =$ (x) $(x^2 \times x^7)^6 =$

Negative or Fractional Index Notation

Indices can be a negative or a fraction. The rules below explain how to use these types of indices:

$a^{-1} = \frac{1}{a}$ This is called the reciprocal of a.

$a^{-n} = \frac{1}{a^n}$

$a^{\frac{1}{2}} = \sqrt{a}$

$a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 1

Find:

(a) 2^{-4} (b) 3^{-2} (c) 5^{-1}

(d) $4^{\frac{1}{2}}$ (e) $8^{\frac{1}{3}}$ (f) $9^{\frac{3}{2}}$

Solution

(a) $2^{-4} = \frac{1}{2^4} = \frac{1}{2 \times 2 \times 2 \times 2} = \frac{1}{16}$

(b) $3^{-2} = \frac{1}{3^2} = \frac{1}{3 \times 3} = \frac{1}{9}$

(c) $5^{-1} = \frac{1}{5}$

(d) $4^{\frac{1}{2}} = \sqrt{4} = 2$

(e) $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

(f) $9^{\frac{3}{2}} = \left(9^{\frac{1}{2}}\right)^3 = 3^3 = 3 \times 3 \times 3 = 27$

Example 2

Find:

(a) $2^{-5} \times 2^{6}$ (b) $m^2 \times m^{-4}$ (c) $\dfrac{3^{-7}}{3^2}$

(d) $(2^8 \times 2^6)^{\frac{1}{2}}$ (e) $(a^2 \times b^{-2})^{-1}$ (f) $\left(\dfrac{m^2}{a}\right)^{-2}$

Solution

(a)
$$\begin{aligned} 2^{-5} \times 2^{6} &= 2^{-5+6} \\ &= 2^1 \\ &= 2 \end{aligned}$$

(b)
$$\begin{aligned} m^2 \times m^{-4} &= m^{2-4} \\ &= m^{-2} \\ &= \frac{1}{m^2} \end{aligned}$$

(c)
$$\begin{aligned} \frac{3^{-7}}{3^2} &= 3^{-7-2} \\ &= 3^{-9} \\ &= \frac{1}{3^9} \end{aligned}$$

(d)
$$\begin{aligned} (2^8 \times 2^6)^{\frac{1}{2}} &= (2^{8+6})^{\frac{1}{2}} \\ &= (2^{14})^{\frac{1}{2}} \\ &= 2^{14 \times \frac{1}{2}} \\ &= 2^7 \end{aligned}$$

(e)
$$\begin{aligned} (a^2 \times b^{-2})^{-1} &= a^{-2} \times b^2 \\ &= \frac{b^2}{a^2} \end{aligned}$$

(f)
$$\begin{aligned} \left(\frac{m^2}{a}\right)^{-2} &= (m^2a^{-1})^{-2} \\ &= m^{-4}a^2 \\ &= \frac{a^2}{m^4} \end{aligned}$$

Skill Exercises: Negative or Fractional Index Notation

1. Find:

(a) $4^{-2} =$ (b) $2^{-3} =$ (c) $6^{-1} =$

(d) $7^{-1} =$ (e) $9^{\frac{1}{2}} =$ (f) $64^{\frac{1}{2}} =$

(g) $16^{\frac{1}{4}} =$ (h) $27^{\frac{1}{3}} =$ (i) $1^{\frac{1}{3}} =$

(j) $5^{-2} =$ (k) $16^{\frac{3}{4}} =$ (l) $4^{\frac{5}{2}} =$

(m) $9^{\frac{7}{2}} =$ (n) $25^{\frac{3}{2}} =$ (o) $8^{-\frac{1}{3}} =$

2. Complete the missing numbers:

(a) $3^{?} = \frac{1}{81}$ (b) $2^{?} = \frac{1}{2}$ (c) $5^{?} = \frac{1}{125}$

(d) $36^{?} = 6$ (e) $36^{?} = \frac{1}{6}$ (f) $7^{?} = 49$

(g) $7^{?} = 343$ (h) $17^{?} = \frac{1}{17}$ (i) $125^{?} = 5$

(j) $\frac{1}{2} = 2^{?}$ (k) $\frac{1}{4} = 2^{?}$ (l) $\frac{1}{100} = 10^{?}$

(m) $\frac{1}{a^3} = a^{?}$ (n) $\sqrt{m} = m^{?}$ (o) $\frac{1}{p^2} = p^{?}$

(p) $\sqrt[3]{q} = q^{?}$ (q) $\sqrt[3]{q^2} = q^{?}$ (r) $\sqrt[5]{q^2} = q^{?}$

3. Use a calculator to find:

(a) 8^{-1} (b) 20^{-1} (c) $\left(\frac{1}{2}\right)^{-1}$

(d) $\left(\frac{1}{4}\right)^{-1}$ (e) 15^{-2} (f) 20^{-3}

(g) $81^{\frac{3}{2}}$ (h) $243^{\frac{3}{5}}$ (i) $16^{-\frac{1}{4}}$

(j) $144^{\frac{3}{2}}$ (k) $169^{\frac{7}{2}}$ (l) $121^{\frac{3}{2}}$

4. Simplify the following expressions so that they contain no negative indices.

(a) $a^6 \times a^{-7} =$ (b) $\frac{a^7}{a^{-3}} =$ (c) $\frac{a^{-5}}{a^{-9}} =$

(d) $a^{-4} \times a^{-2} =$ (e) $(a^2)^{-1} =$ (f) $(a^2)^{-3} =$

(g) $(a^{-2})^{-4} =$ (h) $\left(a^{\frac{1}{2}}\right)^5 =$ (i) $(a^3)^{-\frac{1}{2}} =$

(j) $(a^6)^{\frac{1}{3}} =$ (k) $(a^9)^{-\frac{1}{3}} =$ (l) $(a^{-12})^{\frac{1}{4}} =$

(m) $\left(\frac{a}{b}\right)^2 =$ (n) $(a^2 \times b^{-4})^3 =$ (o) $\left(a^3 b^{\frac{1}{2}}\right)^4 =$

(p) $(a^2 b^{-2})^{-2} =$ (q) $\left(\frac{a^2}{b^3}\right)^4 =$ (r) $(m^{-1} n^3)^{-2} =$

(s) $\left(\frac{a^6}{b^{10}}\right)^{\frac{1}{2}} =$ (t) $\left(\frac{a^2}{m^4}\right)^{-\frac{1}{2}} =$ (u) $\left(\frac{a^8 b^2}{c^6}\right)^{-\frac{1}{2}} =$

(v) $\left(\frac{m^2}{x}\right)^{-1} =$ (w) $\left(\frac{x^2 y}{z^3}\right)^{-4} =$ (x) $\left[(a^3 b^{-8})^{-\frac{1}{3}}\right]^2 =$

5. (a) Express $81^{-\frac{1}{2}}$ as a fraction in the form $\frac{a}{b}$, where a and b are integers.

(b) Simplify $a^6 \div a^2$.

(c) Find the value of y for which $2 \times 4y = 64$.

Section 1.3 Working With Absolute Values

Calculating Absolute Values

The absolute value of a number is its value when the sign is ignored. The absolute value of a number x, is written as $|x|$.

$|3| = 3$ $|{}^{-}3| = 3$

The absolute value of 3 is 3. The absolute value of $^{-}3$ is 3.

Example
Calculate:

(a) $|{}^{-}4|$ (b) $|15|$ (c) $|11 - 7|$ (d) $|9 - 17|$

Solution

(a) 4 (b) 15 (c) 4 (d) 8

Skill Exercises: Calculating Absolute Values

Calculate the following:

(a) $|{}^{-}7|$

(b) $|16|$

(c) $|{}^{-}24|$

(d) $|4 - 5|$

(e) $|{}^{-}4 + 1|$

(f) $|{}^{-}3 + 9|$

(g) $|{}^{-}6 + {}^{-}7|$

(h) $|5 + {}^{-}3|$

(i) $|3 - 4 \times 6|$

(j) $|{}^{-}2| - |{}^{-}7|$

(k) $|6 - 11| + |{}^{-}7| - |5 - 18|$

(l) $|{}^{-}5 + 4 \times {}^{-}8 - 9|$

(m) $|{}^{-}3 \times 5 + {}^{-}6 \times 5|$

(n) $|5 - |{}^{-}4 + 3| - 9|$

(o) $|{}^{-}10 + 3 \times |2 - 7| - 19|$

Unit 2: ALGEBRA — PART 1

In this unit you will be:

2.1 Working with Algebraic Expressions

- Substituting into Formulae.
- Changing the Subject.
- Applying the Four Operations to Algebraic Fractions.

  A. Addition and Subtraction.

  B. Multiplication and Division.

Section 2.1 Working With Algebraic Expressions

Substituting Into Formulae

The process of replacing the letters in a formula with numbers is known as substitution.

Example 1

The length of a metal rod is l. The length changes with temperature and can be found by the formula:

$$l = 40 + 0.02T$$

where T is temperature. Find the length of the rod when:

(a) $T = 50°\text{C}$ (b) $T = {}^{-}10°\text{C}$

Solution

(a) Using $T = 50$ gives:

$$\begin{aligned} l &= 40 + 0.02 \times 50 \\ &= 40 + 1 \\ &= 41 \end{aligned}$$

(b) Using $T = {}^{-}10$ gives:

$$\begin{aligned} l &= 40 + 0.02 \times ({}^{-}10) \\ &= 40 + ({}^{-}0.2) \\ &= 40 - 0.02 \\ &= 39.8 \end{aligned}$$

Example 2

The profit made by a salesman when he makes sales on a day is calculated with the formula:

$$P = 4n - 50$$

Find the profit if he makes:

(a) 30 sales (b) 9 sales

Solution

(a) Here $n = 30$, so the formula gives:

$$\begin{aligned} P &= 40 \times 30 - 50 \\ &= 120 - 50 \\ &= 70 \end{aligned}$$

(b) Here $n = 9$, so the formula gives:

$$\begin{aligned} P &= 4 \times 9 - 50 \\ &= 36 - 50 \\ &= {}^{-}14 \end{aligned}$$

So a loss is made if only 9 sales are made.

Skill Exercises: Substituting into Formulae

1. The formula below is used to convert temperatures in degrees Celsius to degrees Fahrenheit, where F is the temperature in degrees Fahrenheit and C is the temperature in degrees Celsius.

   $F = 1.8C + 32$

   Find F if:

   (a) $C = 10$ (b) $C = 20$ (c) $C = {}^{-}10$

   (d) $C = {}^{-}5$ (e) $C = {}^{-}20$ (f) $C = 15$

2. The formula:

   $$s = \frac{1}{2}(u + v)t$$

   is used to calculate the distance, s, that an object travels if it starts with an initial velocity u and has a velocity v, t seconds later. Find s if:

   (a) $u = 2$, $v = 8$, $t = 2$ (b) $u = 3$, $v = 5$, $t = 10$

   (c) $u = 1.2$, $v = 3.8$, $t = 4.5$ (d) $u = {}^{-}4$, $v = 8$, $t = 2$

   (e) $u = 4$, $v = {}^{-}8$, $t = 5$ (f) $u = 1.6$, $v = 2.8$, $t = 3.2$

3. The length, l, of a spring is given by the formula:

   $l = 20 - 0.08F$

   where F is the size of the force applied to the spring to compress it. Find l if:

   (a) $F = 5$ (b) $F = 20$ (c) $F = 24$ (d) $F = 15$

4. The formula:

   $P = 120n - 400$

   gives the profit, P, made when n cars are sold in a day at a car yard. Find P if:

   (a) $n = 1$ (b) $n = 3$ (c) $n = 4$ (d) $n = 10$

5. Work out the value of each function by substituting the values given.

   (a) $V = p^2 + q^2$
   $p = 8$ and $q = 4$

   (b) $p = a^2 - b^2$
   $a = 10$ and $b = 7$

   (c) $z = \sqrt{x + y}$
   $x = 10$ and $y = 6$

   (d) $Q = \sqrt{x - y}$
   $x = 15$ and $y = 6$

   (e) $P = \dfrac{x + y}{2}$
   $x = 4$ and $y = {}^-10$

   (f) $Q = \sqrt{\dfrac{a}{b}}$
   $a = 100$ and $b = 4$

   (g) $V = \dfrac{x + 2y + z}{5}$
   $x = 2$, $y = {}^-5$ and $z = 8$

   (h) $R = \dfrac{1}{a} + \dfrac{1}{b}$
   $a = 4$ and $b = 2$

   (i) $S = \dfrac{a}{b} + \dfrac{b}{c}$
   $a = 3$, $b = 4$ and $c = 16$

   (j) $R = 0.2a + 0.4b$
   $a = 10$ and $b = 20$

   (k) $T = \dfrac{a}{2} + \dfrac{b}{5}$
   $a = {}^-20$ and $b = 40$

   (l) $C = \dfrac{ab}{a + b}$
   $a = 10$ and $b = {}^-5$

(m) $P = \sqrt{\dfrac{x^2}{y}}$

$x = 10$ and $y = 4$

(n) $A = \dfrac{ab^2}{c}$

$a = 2$, $b = 3$ and $c = 100$

(o) $X = \dfrac{b + c}{a}$

$a = 10$, $b = 1.7$ and $c = 2.1$

(p) $z = \sqrt{x^2 + y^2}$

$x = {}^-3$ and $y = 4$

(q) $P = \sqrt{a^2 - b^2}$

$a = {}^-10$ and $b = 6$

(r) $Q = \sqrt{x^2 + y^2 + z^2}$

$x = {}^-10$, $y = 5$ and $z = 10$

6. Work out the value of each function by substituting the values given.

(a) $P = \dfrac{x - y}{z}$

$x = 10$, $y = 2.02$ and $z = 2.1$

(b) $V = \dfrac{x - y}{x + y}$

$x = 4.9$ and $y = 3.1$

(c) $R = \dfrac{x^2 - y^2}{4}$

$x = 3.6$ and $y = 1.6$

(d) $D = \dfrac{2}{x} + \dfrac{2}{y}$

$x = 0.4$ and $y = 0.8$

(e) $Q = \dfrac{x^2 + y^2}{5}$

$x = 3.7$ and $y = 5.9$

(f) $V = \dfrac{3x + 2y}{x + y}$

$x = 1.6$ and $y = 2.4$

(g) $R = \dfrac{p + q}{p - q}$

$p = 1.2$ and $q = {}^-0.4$

(h) $A = \dfrac{x^2 + y^2}{x + y}$

$x = 5.2$ and $y = {}^-1.2$

(i) $P = \sqrt{x - \dfrac{y}{10}}$

$x = 3.09$ and $y = {}^-106$

7. The formula to convert temperatures from degrees Fahrenheit (°F) into degrees Celsius (°C) is:

   $$C = \frac{5}{9}(F - 32)$$

   Calculate the temperature in degrees Celsius that is equivalent to a temperature of $^{-}7$°F.

8. $F = \dfrac{9R}{4} + 32$

   Calculate the value of F when $R = {}^{-}20$.

9. Given that $m = \frac{1}{2}$, $p = \frac{3}{4}$, $t = {}^{-}2$, calculate:

   (a) $mp + t$ (b) $\dfrac{(m + p)}{t}$

Changing the Subject

Sometimes a formula can be rearranged into a more useful format. For example, the formula:

$$F = 1.8C + 32$$

can be used to convert temperatures in degrees Celsius to degrees Fahrenheit. It can be rearranged to enable temperature in degrees Fahrenheit to be converted to degrees Celsius. We say that the formula has been rearranged to make C the subject of the formula.

Example 1
Rearrange the formula:

$$F = 1.8C + 32$$

to make C the subject of the formula.

Solution

The aim is to remove all terms from the right hand side of the equation except for the C.

First subtract 32 from both sides which gives $F - 32 = 1.8C$

Then dividing both sides by 1.8 gives $\dfrac{F - 32}{1.8} = C$

Therefore, the formula can be rearranged as $C = \dfrac{F - 32}{1.8}$

Example 2

Make v the subject of the formula:

$$s = \frac{(u + v)t}{2}$$

Solution

First multiply both sides of the formula by 2 to give: $2s = (u + v)t$

Then divide both sides by t, to give: $\frac{2s}{t} = u + v$

Finally subtract u from both sides to give: $\frac{2s}{t} - u = v$

Therefore, the formula becomes: $v = \frac{2s}{t} - u$

Skill Exercises: Changing the Subject

1. Make x the subject of each of the following formulae:

   (a) $y = 4x$
   (b) $y = 2x + 3$
   (c) $y = 4x - 8$
   (d) $y = \frac{x + 2}{4}$
   (e) $y = \frac{x - 2}{5}$
   (f) $y = x + a$
   (g) $y = \frac{x - b}{a}$
   (h) $y = ax + c$
   (i) $y = \frac{ax + b}{c}$
   (j) $y = \frac{ax - c}{b}$
   (k) $y = a + b + x$
   (l) $y = \frac{x - a + b}{c}$
   (m) $y = abx$
   (n) $y = abx + c$
   (o) $y = \frac{4ax - b}{3c}$
   (p) $p = \frac{ax - bc}{d}$
   (q) $y = (a + x)b$
   (r) $y = \frac{(3 + x)a}{4}$
   (s) $q = \frac{3(x - 4)}{2}$
   (t) $v = \frac{5(x + y)}{4}$
   (u) $z = \frac{(x - 3)}{4} + a$

2. Ohm's law is used in electrical circuits and states that:

   $V = IR$

   Write formulae with I and R as their subjects.

3. Newton's Second law states that $F = ma$. Write formulae with m and a as their subjects.

4. The formula $C = 2\pi r$ can be used to find the circumference of a circle. Make r the subject of this formula.

5. The equation $v = u + at$ is used to find the velocities of objects.
   (a) Make t the subject of this formula.
   (b) Make a the subject of this formula.

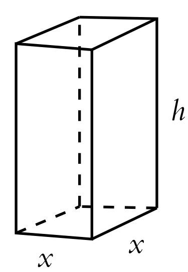
6. The mean of three numbers x, y and z can be found using the formula:
   $$m = \frac{x + y + z}{3}$$
   Make z the subject of this formula.

7. Make a the subject of the following formula:
   (a) $v^2 = u^2 + 2as$

8. The formula $V = xyz$ can be used to the find the volume of a rectangular box. Make z the subject of the formula.

9. The volume of a can is given by $V = \pi r^2 h$ where r is the radius of the base and h is the height of the can.
   (a) Make r the subject of the equation.
   (b) Find r correct to 2 decimal places if $V = 250$ cm^3 and $h = 10$ cm.

10. A box with a square base has its volume given by $V = x^2h$ and its surface area given by $A = 2x^2 + 4xh$.
   (a) Make h the subject of both formulae.
   (b) Find h if $A = 24$ cm^2 and $x = 2$ cm.
   (c) Find h if $V = 250$ cm^3 and $x = 10$ cm.



Applying the Four Operations to Algebraic Fractions

A. Addition and Subtraction

When fractions are added or subtracted, a common denominator must be used. To find a common denominator multiply the two denominators together and form equivalent fractions.

$$\frac{1}{2}+\frac{1}{3} = \frac{3}{6}+\frac{2}{6} \quad \text{(common denominator is } 2 \times 3 = 6\text{)}$$

$$= \frac{5}{6}$$

When working with algebraic fractions, a common denominator must also be used.

Example 1

Simplify:

$$\frac{x}{6}+\frac{x}{5}$$

Solution

These fractions are added by using a common denominator of 30. ($6 \times 5 = 30$).

$$\frac{x}{6}+\frac{x}{5} = \frac{5x}{30} + \frac{6x}{30}$$

$$= \frac{11x}{30}$$

Example 2

Simplify:

$$\frac{3}{x}-\frac{2}{x+1}$$

Solution

The common denominator is $x \times (x + 1) = x(x + 1)$.
Therefore,

$$\frac{3}{x}-\frac{2}{x+1} = \frac{3(x+1)}{x(x+1)} - \frac{2x}{x(x+1)}$$

$$= \frac{3x+3}{x(x+1)} - \frac{2x}{x(x+1)}$$

$$= \frac{x+3}{x(x+1)}$$

Skill Exercises: Addition and Subtraction

1. Simplify each expression into a single fraction:

(a) $\frac{x}{4}+\frac{x}{5}=$ (b) $\frac{x}{7}+\frac{x}{4}=$ (c) $\frac{x}{3}+\frac{x}{5}=$

(d) $\frac{2y}{7}+\frac{5y}{3}=$ (e) $\frac{2y}{5}+\frac{3y}{4}=$ (f) $\frac{5y}{7}+\frac{8y}{7}=$

(g) $\frac{4x}{7}-\frac{3x}{10}=$ (h) $\frac{5x}{6}-\frac{2x}{3}=$ (i) $\frac{x}{4}+\frac{7x}{8}=$

(j) $\frac{5x}{6}+\frac{7x}{24}=$ (k) $\frac{a}{4}+\frac{b}{5}=$ (l) $\frac{x}{3}+\frac{y}{8}=$

(m) $\frac{a}{3}-\frac{b}{5}=$ (n) $\frac{2a}{3}+\frac{4b}{5}=$ (o) $\frac{8a}{9}-\frac{3b}{4}=$

2. Express the following as single fractions:

(a) $\frac{4}{x}+\frac{2}{y}=$ (b) $\frac{6}{x}-\frac{1}{y}=$ (c) $\frac{1}{x}+\frac{3}{y}=$

(d) $\frac{8}{a}-\frac{3}{a}=$ (e) $\frac{4}{a}+\frac{3}{2b}=$ (f) $\frac{5}{3a}-\frac{1}{2b}=$

(g) $\frac{5}{3a}+\frac{4}{5b}=$ (h) $\frac{7}{3a}-\frac{4}{5a}=$ (i) $\frac{6}{7a}-\frac{1}{4a}=$

(j) $\frac{7}{8a}-\frac{2}{3b}=$ (k) $\frac{3}{6a}-\frac{5}{12a}=$ (l) $\frac{7}{4a}-\frac{3}{8a}=$

3. Combine the fractions below into a single fraction:

(a) $\frac{1}{x}+\frac{1}{x+1}=$ (b) $\frac{2}{x}+\frac{1}{x+2}=$ (c) $\frac{4}{x+1}+\frac{3}{x}=$

(d) $\frac{5}{x}-\frac{1}{x+2}=$ (e) $\frac{5}{x-2}-\frac{1}{x}=$ (f) $\frac{6}{x+3}-\frac{4}{3x}=$

(g) $\frac{6}{x+1}-\frac{6}{x}=$ (h) $\frac{4}{x-5}+\frac{2}{x}=$ (i) $\frac{7}{5x}-\frac{4}{x+6}=$

(j) $\frac{5}{x-7}+\frac{7}{2x}=$ (k) $\frac{6}{x-10}+\frac{5}{3x}=$ (l) $\frac{1}{3x}+\frac{2}{x-8}=$

Applying the Four Operations to Algebraic Fractions

B. Multiplication and Division

When fractions are multiplied, the numerators and denominators are multiplied. The answer is simplified.

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

$$= \frac{1}{2}$$

Example 1

Simplify:

$$\frac{4x}{y} \times \frac{x}{2y}$$

Solution

$$\frac{4x}{y} \times \frac{x}{2y} = \frac{4x^2}{2y^2}$$

$$= \frac{2x^2}{y^2}$$

When fractions are divided, turn the divisor (the second fraction) upside down. Then multiply.

$$\frac{2}{3} \div \frac{3}{4} = \frac{2}{3} \times \frac{4}{3}$$

$$= \frac{8}{9}$$

Example 2
Simplify:

$$\frac{4x^2}{3y} \div \frac{x}{2y^2}$$

Solution

$$\begin{aligned}\frac{4x^2}{3y} \div \frac{x}{2y^2} &= \frac{4x^2}{3y} \times \frac{2y^2}{x} \\ &= \frac{8x^2y^2}{3xy} \\ &= \frac{8xy}{3}\end{aligned}$$

Skill Exercises: Multiplication and Division

1. (a) $\frac{a}{b} \times \frac{a}{b} =$ (b) $\frac{a^2}{3b} \times \frac{2a}{b} =$ (c) $\frac{3a^2}{b} \times \frac{4a}{b^2} =$

 (d) $\frac{a^2}{3b} \times \frac{4a}{b^2} =$ (e) $\frac{x}{w} \times \frac{x^2}{w^2} =$ (f) $\frac{2x^2}{4a} \times \frac{3x}{2b} =$

 (g) $\frac{5b^2}{4x} \times \frac{2b^2}{10x} =$ (h) $\frac{2x^3}{3a} \times \frac{ax}{6a} =$ (i) $\frac{2a}{b} \times \frac{5}{6a^2b} =$

2. (a) $\frac{2a}{b} \div \frac{a}{b} =$ (b) $\frac{3a^2}{x} \div \frac{a}{x} =$ (c) $\frac{a}{b} \div \frac{a}{b} =$

 (d) $\frac{3ax^2}{4} \div \frac{4x}{3} =$ (e) $\frac{3ax^2}{4} \div \frac{4x}{3} =$ (f) $\frac{6a^2b}{5x} \div \frac{2a}{5x} =$

 (g) $\frac{5d}{c} \div \frac{d^2}{3c} =$ (h) $\frac{7b^2c}{3} \div \frac{14b}{7} =$ (i) $\frac{6xa^2}{5c} \div \frac{3x}{5c} =$

Unit 3: ALGEBRA — PART 2

In this unit you will be:

3.1 Working with Graphs and Equations

- Plotting Co-ordinates.
- Drawing Straight Line Graphs.
- Solving Linear Equations 1.
- Listing Domains and Ranges.
- Solving Simultaneous Equations Using Graphs.
- Solving Simultaneous Equations Using Algebra.
- Simplifying Algebraic Equations.
- Solving Linear Equations 2.
- Solving Quadratic Equations by Factorisation.
- Plotting Quadratic Functions.

Section 3.1 Working With Graphs And Equations

Plotting Co-ordinates

Example 1

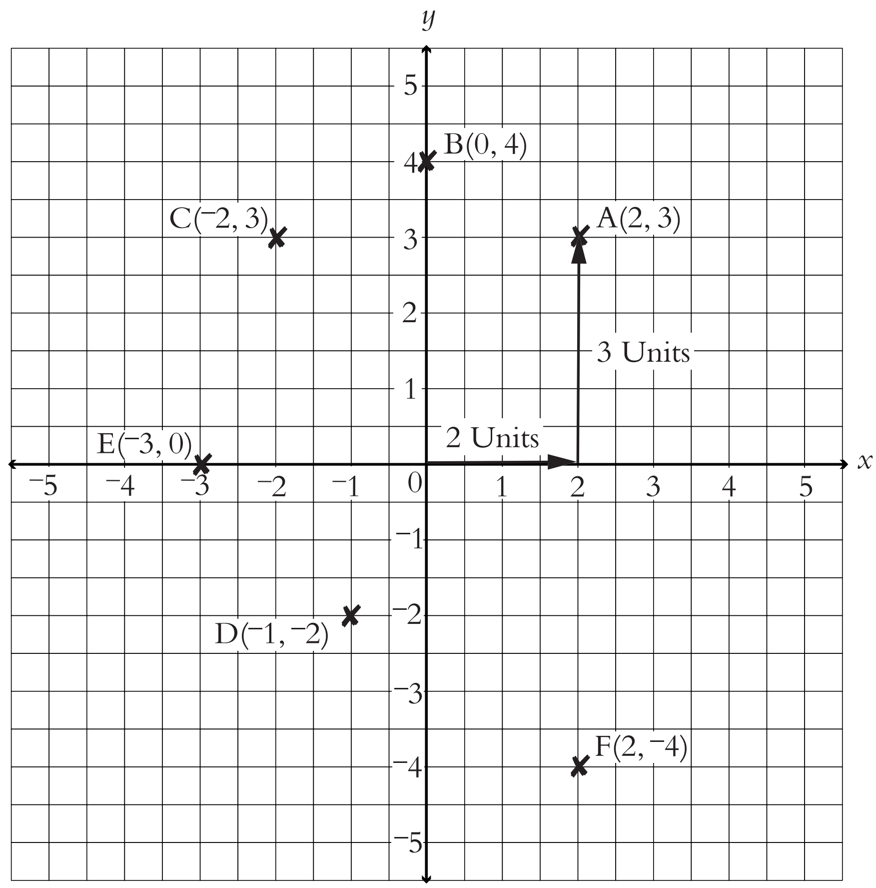
On a set of co-ordinates axes, plot the points:

A(2, 3) B(0, 4) C(⁻2, 3) D(⁻1, ⁻2) E(⁻3, 0) F(2, ⁻4)

Solution

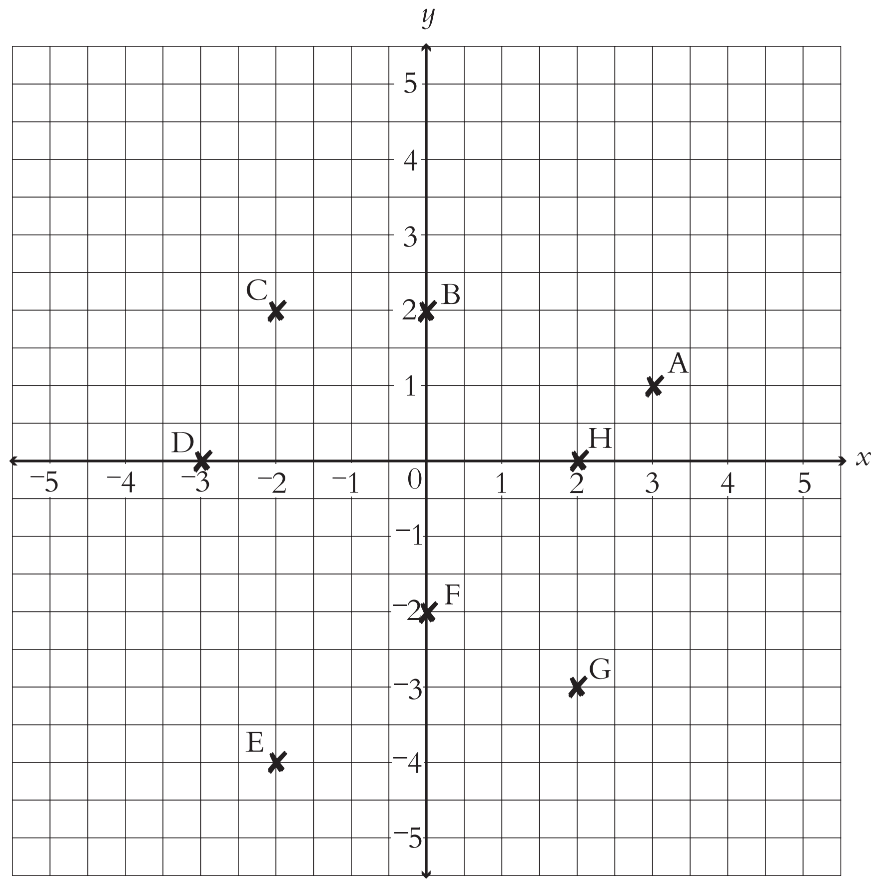
The x-axis and the y-axis cross at the origin (0, 0).

To locate the point A(2, 3), go 2 units horizontally from the origin in the positive x-direction and then 3 units vertically in the positive y-direction, as shown in the diagram.



Example 2

Identify the co-ordinates of the points A, B, C, D, E, F, G and H shown on the following grid:

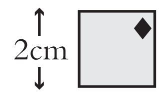


Solution

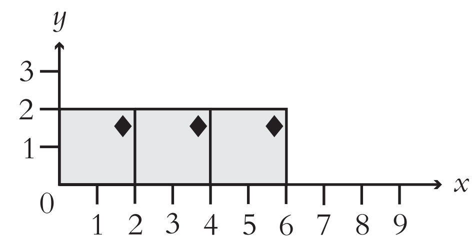
A(3, 1), B(0, 2), C(−2, 2), D(−3, 0), E(−2, −4), F(0, −2), G(2, −3), H(2, 0)

Example 3

Makara has ten square tiles like this:



Makara places all the square tiles in a row. He starts his row like this:



For each square tile he writes down the co-ordinates of the corner that has a ♦.

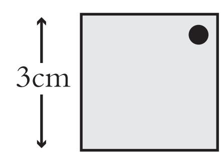
The co-ordinates of the first corner are (2, 2).

(a) Write down the co-ordinates of the next five corners which have a ♦.

(b) Look at the numbers in the co-ordinates. Describe two things you notice.

(c) Makara thinks that (17, 2) are the co-ordinates of one of the corners which have a ♦. Explain why he is wrong.

(d) Sinapi has some bigger square tiles, like this:



She places them next to each other in a row, like Makara's tiles.

Write down the co-ordinates of the first two corners that have a ●.

Solution

(a) (4, 2), (6, 2), (8, 2), (10, 2), (12, 2)

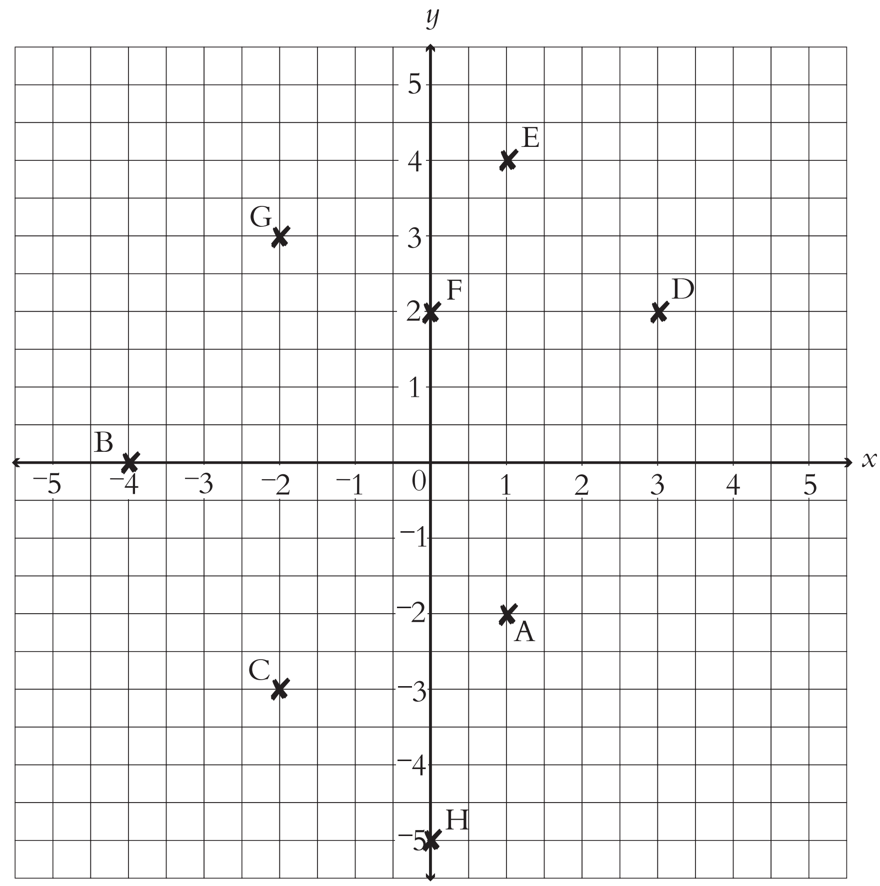
(b) The x-co-ordinate increases by 2 each time; the y-co-ordinate remains constant at 2.

(c) (17, 2) cannot be the co-ordinates of a corner as 17 is an odd number and the corners that have a ♦ all have even co-ordinates.

(d) (3, 3), (6, 3)

Skill Exercises: Plotting Co-ordinates

1. Write down the co-ordinates of the points marked on the following grid:



2. On a set of co-ordinate axes, with x values from $^-5$ to 5 and y values from $^-5$ to 5, plot the following points:

   A(2, 4), B(1, 2), C($^-2$, 5), D($^-3$, $^-3$), E($^-2$, $^-4$), F(0, $^-3$), G($^-4$, 0), H(2, $^-3$)

   What can you say about A, B and E?

3. On a suitable set of co-ordinate axes, join the points (3, 0), (0, 4) and ($^-3$, 0). What shape have you made?

4. Three corners of a square have co-ordinates (4, 2), ($^-2$, 2) and (4, $^-4$). Plot these points on a grid and state the co-ordinates of the other corner.

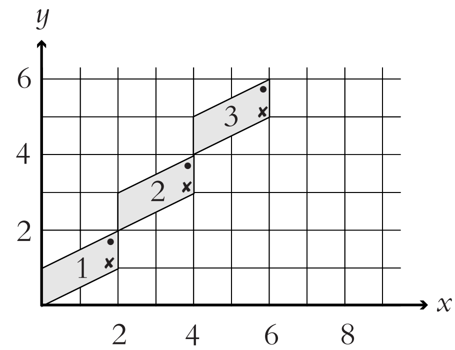
5. Three corners of a rectangle have co-ordinates (4, 1), ($^-2$, 1) and ($^-2$, $^-3$). Plot these points on a grid and state the co-ordinates of the other corner.

6. Two adjacent corners of a square have co-ordinates ($^-1$, 1) and (2, 1).

   (a) What is the length of a side of the square?

   (b) What are the possible co-ordinates of the other two points?

7. Siaki has some parallelogram tiles. He puts them on a grid, in a continuing pattern. He numbers each tile. The diagram shows part of the pattern of tiles on the grid.



Siaki marks the top right corner of each tile with a ●. The co-ordinates of the corner with a ● on tile number 3 are (6, 6).

(a) What are the co-ordinates of the corner with a ● on tile number 4?

(b) What are the co-ordinates of the corner with a ● on tile number 20? Explain how you worked out your answer.

(c) Siaki says, 'One tile in the pattern has a ● in the corner at (25, 25).' Explain why Siaki is wrong.

Tile number	Co-ordinates of the corner with a ✘
1	(2, 1)
2	
3	
4	

(d) Siaki marks the bottom right corner of each tile with a ✘. Copy and complete the table to show the co-ordinates of each corner with a ✘.

(e) Copy and complete the statement:

'Tile number 7 has a ✘ in the corner at (____, ____).'

(f) Copy and complete the statement:

'Tile number ____ has a ✘ in the corner at (20, 19).'

8. A robot can move about on a grid. It can move North, South, East or West. It must move one step at a time. The robot starts from the point marked ● on the grid below:

It takes 2 steps.

1st step:	West
2nd step:	North

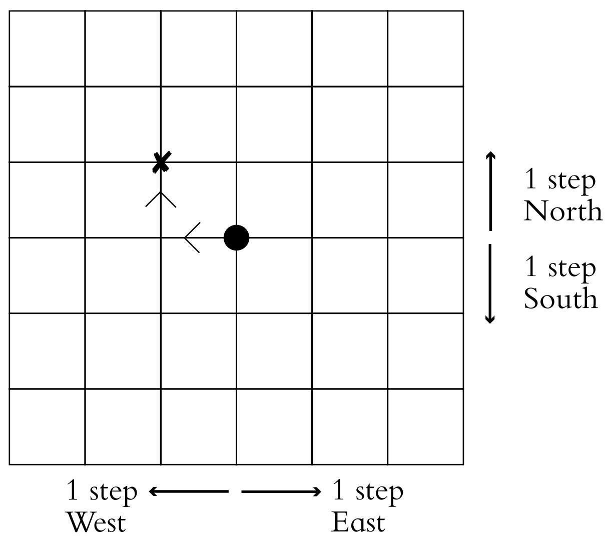
It gets to point marked ✘.

(a) The robot starts again from the point marked ●.

It takes 2 steps.

1st step:	South
2nd step:	South

Copy the grid below and mark the point it gets to with a ✘.

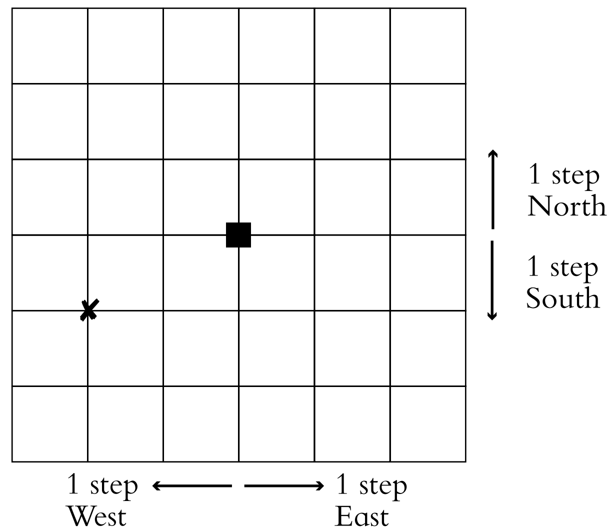


(b) The robot always starts from the point marked ●.

Find all the points the robot can reach in 2 steps.

Mark each point with a ✘ on the grid you have drawn.

(c) Another robot always starts from the point marked ■ on this grid.



It takes 3 steps.

1st step:	South
2nd step:	West
3rd step:	West

It gets to the point marked ✘.

The robot starts again from the point marked ■.

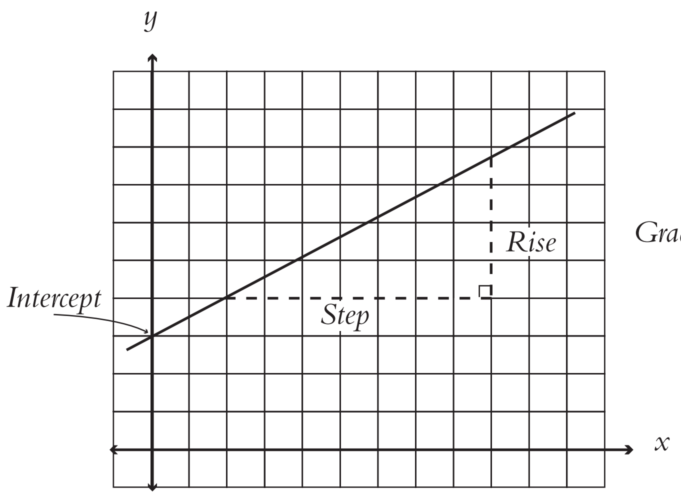
Copy and complete the table to show two more ways for the robot to get to the point marked ✘ in 3 steps.

1st step	South	West	
2nd step	West		
3rd step	West		

Drawing Straight Line Graphs

This section looks at how to calculate co-ordinates and how to plot straight line graphs. It also looks at the gradient and intercept of a straight line and the equation of a straight line.

The gradient of a line is a measure of its steepness. The intercept of a line is the value where the line crosses the y-axis.



$$\textit{Gradient} = \frac{\textit{Rise}}{\textit{Step}}$$

The equation of a straight line is $y = mx + c$, where m = gradient and c = intercept (where the line crosses the y-axis).

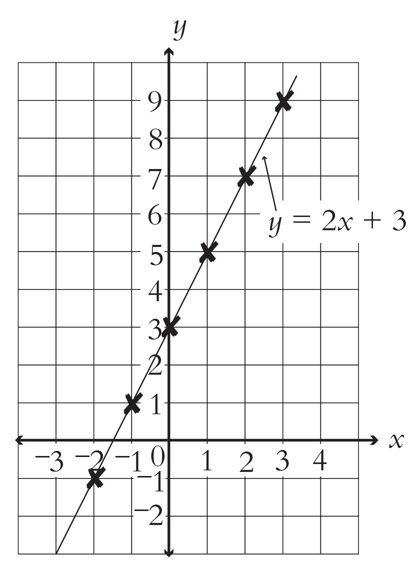
Example 1
Draw the graph with equation $y = 2x + 3$.

Solution

First, find the co-ordinates of some points on the graph. This can be done by calculating y for a range of x values as shown in the table.

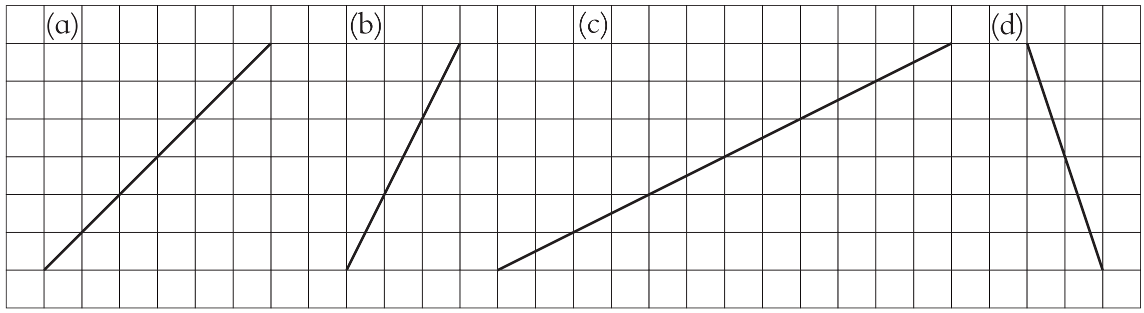
x	−2	−1	0	1	2	3
y	−1	1	3	5	7	9

The points can then be plotted on a set of axes and a straight line drawn through them.



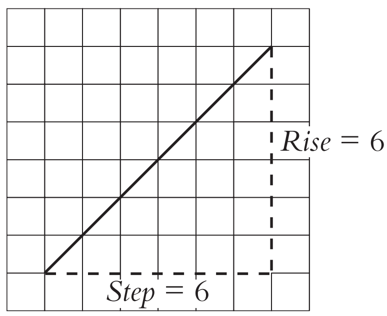
Example 2

Calculate the gradient of each of the following lines:



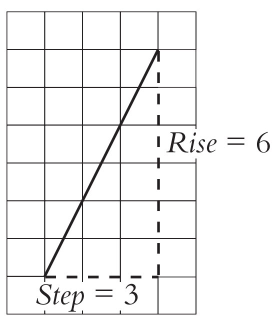
Solution

(a)



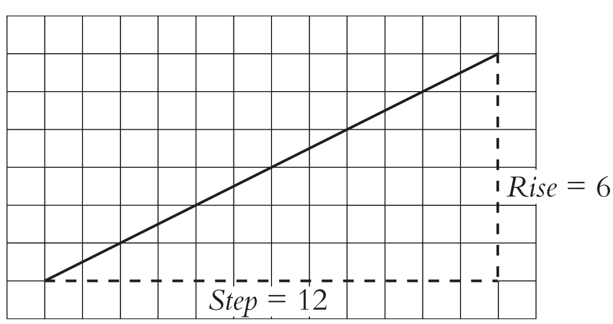
$$\text{Gradient} = \frac{6}{6}$$
$$= 1$$

(b)



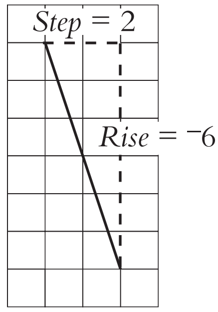
$$\text{Gradient} = \frac{6}{3}$$
$$= 2$$

(c)



$$\text{Gradient} = \frac{6}{12}$$
$$= \frac{1}{2}$$

(d)

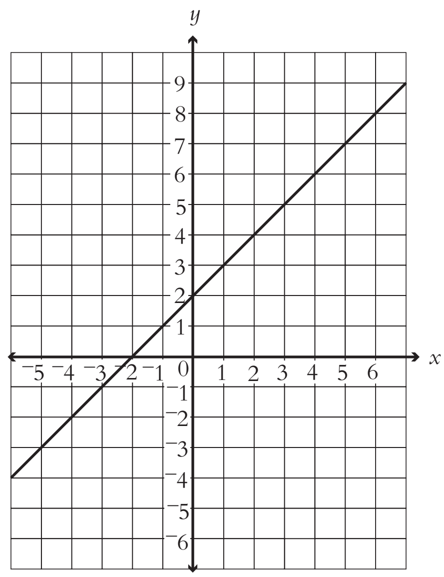


$$\text{Gradient} = \frac{^-6}{2}$$
$$= {}^-3$$

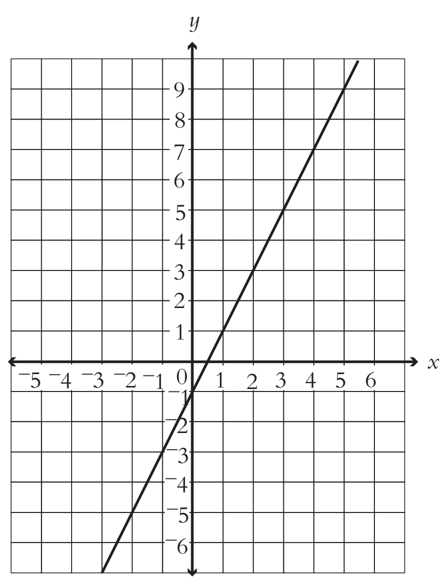
Example 3

Determine the equation of each of the following lines:

(a)

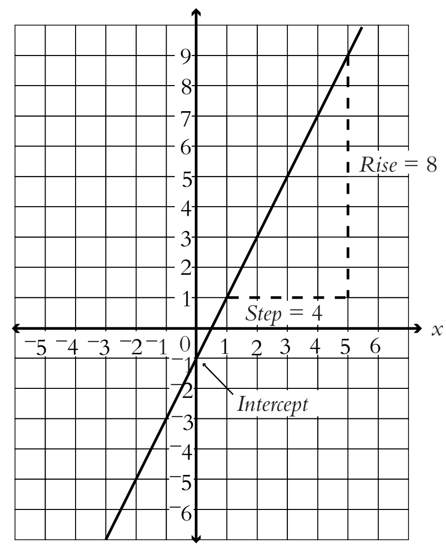


(b)



Solution

(a)



$$\text{Gradient} = \frac{8}{4}$$

$$= 2$$

Intercept = 2

So $m = 1$ and $c = 2$

The equation is:

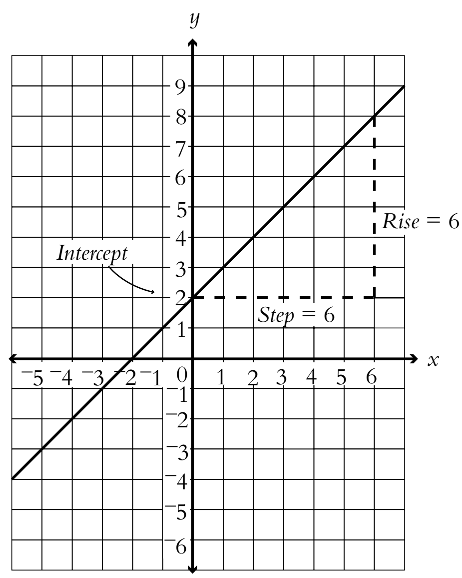
$y = mx + c$

$y = 1x + 2$

or

$y = x + 2$

(b)



$$\text{Gradient} = \frac{6}{6}$$

$$= 1$$

y Intercept = $^{-}1$

So $m = 2$ and $c = {}^{-}1$.

The equation is:

$y = mx + c$

$y = 2x + ({}^{-}1)$

or $y = 2x - 1$.

Skill Exercises: Drawing Straight Line Graphs

1. (a) Copy and complete the following table for $y = 2x - 2$.

x	$^-2$	$^-1$	0	1	3	5
y						

(b) Draw the graph of $y = 2x - 2$.

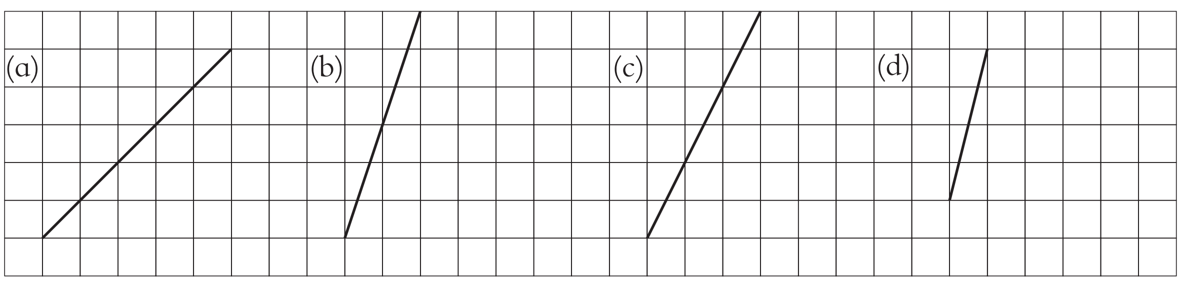
2. Draw the graphs with equations given below, using a new set of axes for each graph.

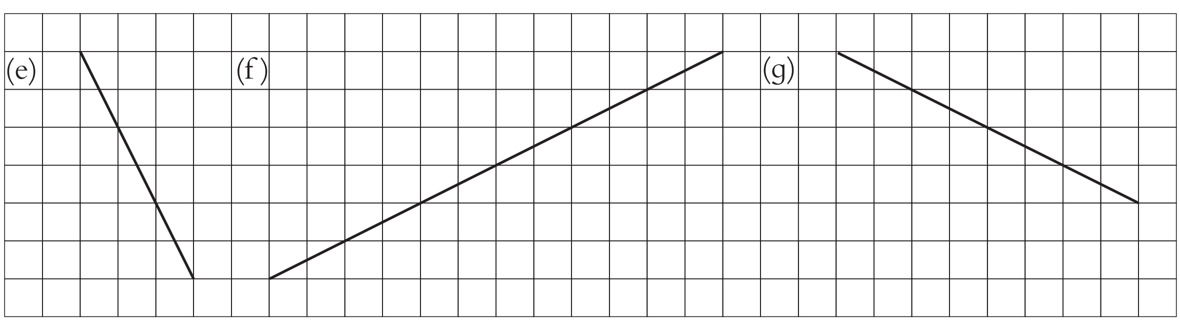
(a) $y = x + 3$ (b) $y = x - 4$

(c) $y = 4x - 1$ (d) $y = 3x + 1$

(e) $y = 4 - x$ (f) $y = 8 - 2x$

3. Calculate the gradient of each of the following lines, (a) to (g):





4. Write down the equations of the lines with gradients and intercepts listed below:

   (a) Gradient = 4 and intercept = 2.

   (b) Gradient = 2 and intercept = $^-5$.

   (c) Gradient = $\frac{1}{2}$ and intercept = 1.

   (d) Gradient = $^-1$ and intercept = $^-5$.

5. Copy and complete the following table, which gives the equation, gradient and intercept for a number of straight lines.

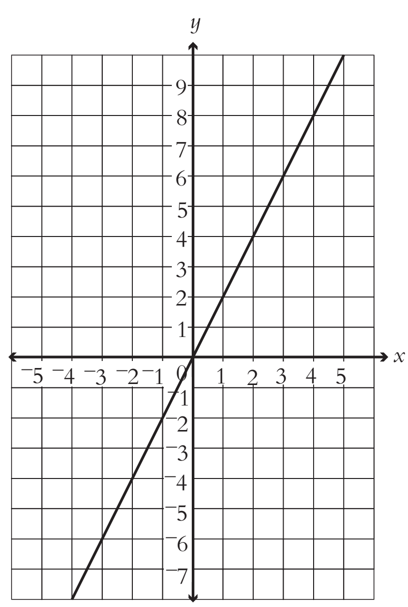
Equation	Gradient	Intercept
$y = 5x + 7$		
	3	$^-2$
$y = {}^-3x + 2$		
$y = {}^-4x - 2$		
	$^-2$	3
	$\frac{1}{2}$	1
$y = 4 - x$		
$y = 10 - 3x$		

6. (a) Plot the points A, B and C with co-ordinates:
   A(2, 4) B(7, 5) C(0, 10)
   and join them to form a triangle.

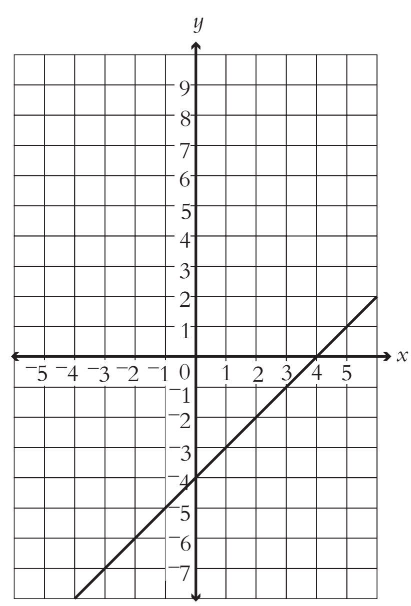
   (b) Calculate the gradient of each side of the triangle.

7. Determine the equation of each of the following lines:

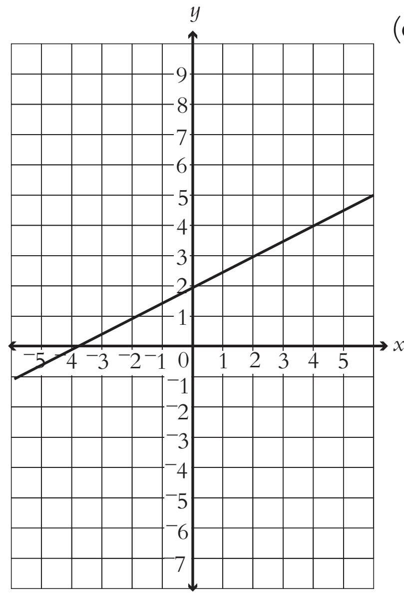
(a)



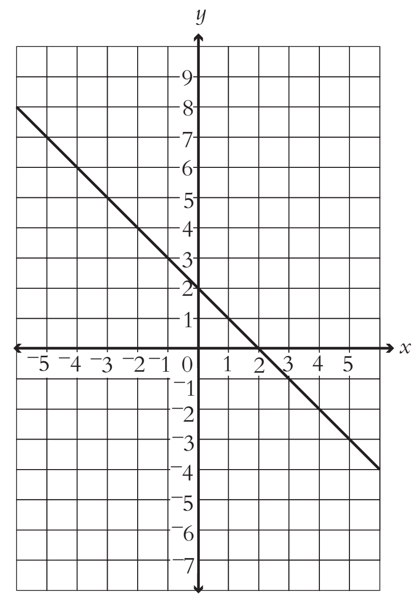
(b)



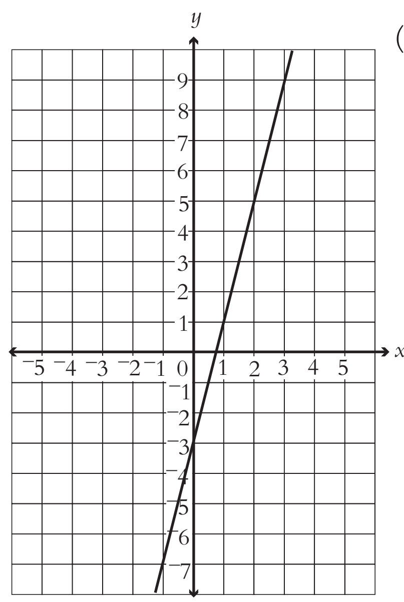
(c)



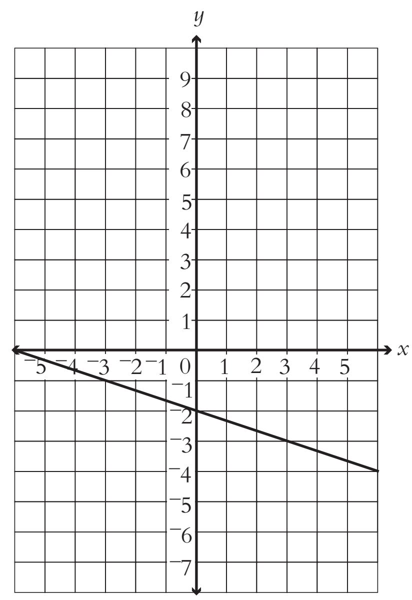
(d)



(e)



(f)



8. (a) On a set of axes, plot the points with co-ordinates:

   $(^-2, ^-2)$ $(2, 0)$ $(4, 1)$ $(6, 2)$

   and then draw a straight line through these points.

   (b) Determine the equation of the line.

9. (a) On the same axes, draw the lines with equation $y = 2x + 3$ and $y = 8 - \frac{1}{2}x$.

   (b) Write down the co-ordinates of the point where the lines cross.

10. The point A has co-ordinates (4, 2), the point B has co-ordinates (8, 6) and the point C has co-ordinates (5, 9).

    (a) Plot these points on a set of axes and draw straight lines through each point to form a triangle.

    (b) Work out the equation of each of the line you have drawn.

Solving Linear Equations 1

Example 1

Solve the following equations:

(a) $x + 6 = 13$ (b) $x - 7 = 11$ (c) $4x = 72$ (d) $\frac{x}{3} = 11$

Solution

(a)
$$\begin{aligned} x + 6 &= 13 \\ x &= 13 - 6 && \text{(subtract 6 from both sides)} \\ x &= 7 \end{aligned}$$

(b)
$$\begin{aligned} x - 7 &= 11 \\ x &= 11 + 7 && \text{(add 7 to both sides)} \\ x &= 18 \end{aligned}$$

(c)
$$\begin{aligned} 4x &= 72 \\ x &= \frac{72}{4} && \text{(divide both sides by 4)} \\ x &= 18 \end{aligned}$$

(d)
$$\begin{aligned} \frac{x}{3} &= 11 \\ x &= 11 \times 3 && \text{(multiply both sides by 3)} \\ x &= 33 \end{aligned}$$

Example 2

Solve the following equations:

(a) $2x + 4 = 20$ (b) $\frac{x + 4}{6} = 3$ (c) $4(x + 4) = 18$

Solution

(a)
$$\begin{aligned} 2x + 4 &= 20 \\ 2x &= 20 - 4 && \text{(subtract 4 from both sides)} \\ 2x &= 16 \\ x &= \frac{16}{2} && \text{(divide both sides by 2)} \\ x &= 8 \end{aligned}$$

(b)
$$\begin{aligned} \frac{x + 4}{6} &= 3 \\ x + 4 &= 3 \times 6 && \text{(multiply both sides by 6)} \\ x + 4 &= 18 \\ x &= 18 - 4 && \text{(subtract 4 from both sides)} \\ x &= 14 \end{aligned}$$

(c)
$$\begin{aligned} 4(x + 4) &= 18 \\ 4x + 16 &= 18 && \text{(remove brackets)} \\ 4x &= 18 - 16 && \text{(subtract 16 from both sides)} \\ 4x &= 2 \\ x &= \frac{2}{4} && \text{(divide both sides by 4)} \\ x &= \frac{1}{2} \end{aligned}$$

Example 3

Solve the following equations:

(a) $4x + x = 3x + 5$ (b) $4x - 4 = 10 - 3x$

Solution

(a)
$$\begin{aligned} 4x + x &= 3x + 5 \\ x + 2 &= 5 && \text{(subtract } 3x \text{ from both sides)} \\ x &= 5 - 2 && \text{(subtract 2 from both sides)} \\ x &= 3 \end{aligned}$$

(b)
$$\begin{aligned} 4x - 4 &= 10 - 3x \\ 7x - 4 &= 10 && \text{(add } 3x \text{ to both sides)} \\ 7x &= 10 + 4 && \text{(add 4 to both sides)} \\ 7x &= 14 \\ x &= \frac{14}{7} && \text{(divide both sides by 7)} \\ x &= 2 \end{aligned}$$

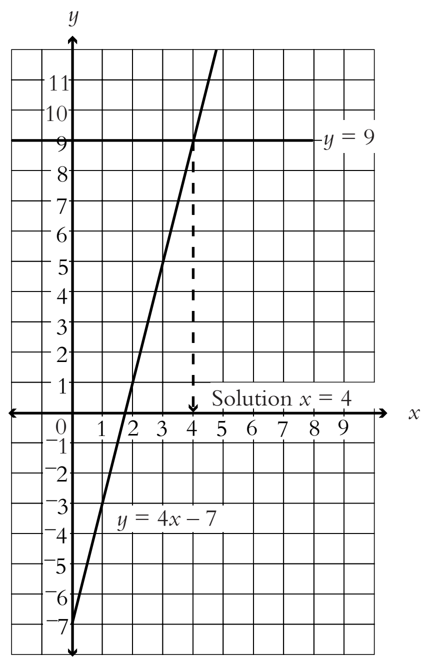
Example 4

Use graphs to solve the following equations:

(a) $4x - 7 = 9$ (b) $x + 7 = 3x - 3$

Solution

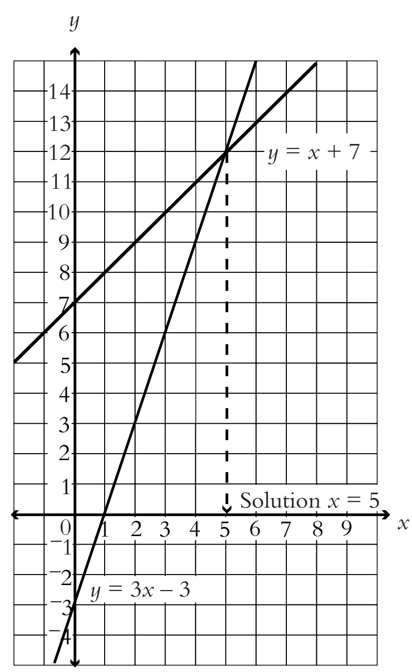
(a) Draw the line $y = 4x - 7$ and $y = 9$



The solution is given by the value on the x-axis immediately below the point where $y = 4x - 7$ and $y = 9$ cross.

The solution is $x = 4$.

(b) Draw the lines $y = x + 7$ and $y = 3x - 3$.



The lines cross where $x = 5$, so this is the solution for the equation.

Skill Exercises: Solving Linear Equations 1

1. Solve the following equations:

(a) $x + 6 = 14$ (b) $x - 3 = 8$ (c) $7x = 21$

(d) $\frac{x}{3} = 10$ (e) $10x = 80$ (f) $5x = 35$

(g) $x + 9 = 22$ (h) $x - 4 = 3$ (i) $x - 22 = 18$

(j) $\frac{x}{5} = 100$ (k) $3x = 96$ (l) $x + 22 = 47$

2. Solve the following equations:

(a) $2x + 7 = 15$ (b) $5x - 3 = 32$ (c) $6x + 4 = 22$

(d) $11x - 3 = 19$ (e) $5x + 2 = 37$ (f) $\frac{x+4}{3} = 21$

(g) $\frac{2x-1}{3} = 5$ (h) $4(x + 2) = 28$ (i) $3(5x - 6) = 147$

(j) $2(3x - 7) = 46$ (k) $\frac{2(x+6)}{3} = 6$ (l) $5(2x + 3) = 35$

3 (a) $x + 1 = 2x - 1$ (b) $2x + 4 = 3x - 1$

(c) $7x - 2 = 5x + 6$ (d) $4x + 7 = 10x - 11$

(e) $x + 18 = 9x - 22$ (f) $7x + 1 = 3x + 17$

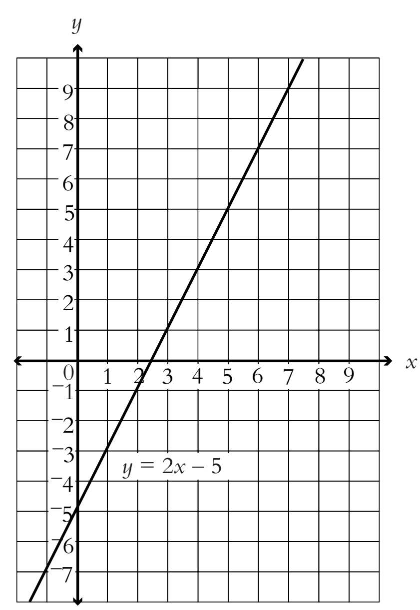
(g) $6(x + 1) = 14(x - 1)$ (h) $2(5x + 3) = 12x - 3$

4. The graph $y = 2x - 5$ is shown below. Use the graph to solve the equations:

(a) $2x - 5 = 1$

(b) $2x - 5 = 7$

(c) $2x - 5 = {}^{-}3$



5. Solve the equation $2x - 3 = 9$ by drawing the graphs $y = 2x - 3$ and $y = 9$.

6. Use a graph to solve the equation $4x - 5 = 3$.

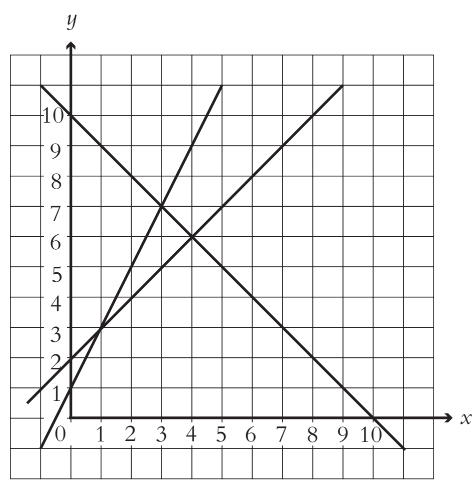
7. (a) On the same set of axes, draw the lines with equations $y = x - 1$ and $y = 2x - 3$.

   (b) Use the graph to find the solution of the equation $x + 1 = 2x - 3$.

8. Use a graph to solve the following equations:

   (a) $2x = {}^{-}x + 3$ (b) $4 - 2x = 2x - 8$

9. The following graph shows the lines with equations $y = 2x + 1$, $y = x + 2$ and $y = 10 - x$.



   Use the graph to solve the equations:

   (a) $2x + 1 = 10 - x$

   (b) $x + 2 = 10 - x$

   (c) $2x + 1 = x + 2$

10. On the same set of axes, draw the graphs of three straight lines and use them to solve the equations:

   (a) $2x - 2 = x + 3$

   (b) $2x - 2 = 8$

   (c) $x + 3 = 8$

Listing Domains and Ranges

Co-ordinates are also called ordered pairs. This means that the order of the co-ordinates is important.

- The x co-ordinate always comes first.
- The y co-ordinate always comes second.

(2, 3) is a different point to (3, 2)

The first values, the x values, are called 'the domain'. The second values, the y values, are called 'the range'.

Example 1
For the set of ordered pairs

{(2, 1), (3, 7), (2, 5), (5, 9), (10, 10)}

list the domain and the range.

Solution

The domain is {2, 3, 5, 10}. Notice that the 2 is only written once.

The range is {1, 5, 7, 9, 10}. Notice that the values have been put in order.

Example 2
A relation is give by $y = 3x - 2$. The domain is $\{^{-}1, 0, 1, 2\}$. Find the range.

Solution

Draw a table. Write in the domain values along the first row. Solve the equation for each value.

x (domain)	$^{-}1$	0	1	2
$3x$	$^{-}3$	0	3	6
$^{-}2$	$^{-}2$	$^{-}2$	$^{-}2$	$^{-}2$
y (range)	$^{-}5$	$^{-}2$	1	4

The range is $\{^{-}5, ^{-}2, 1, 4\}$.

Skill Exercises: Listing Domains and Ranges

1. For each set of ordered pairs, list the domain and range.

   (a) $\{(1, 2), (3, 4), (5, 3)\}$ (b) $\{(1, 5), (2, 5), (3, 7), (4, 7)\}$

   (c) $\{(2, 3), (3, 4), (4, 5), (5, 6), (6, 7)\}$ (d) $\{(^-1, 4), (^-2, 4), (^-3, 4)\}$

2. Complete the table:

Relation	Domain	Range
$y = \frac{1}{2}x$	$\{2, 4, 6, 8\}$	
$y = 3x$		$\{21, 27, 33, 45\}$
$y = x + 2$	$\{0, 1, 2, 3, 4\}$	
	$\{0, 1, 2\}$	$\{5, 6, 7\}$
$y = 2x - 1$	$\{^-2, ^-1, 0, 1, 2\}$	

Solving Simultaneous Equations Using Graphs

Simultaneous equations are two or more equations that are true at the same time. Think about the following example:

Ana and Tala are sisters; we know that:

(i) Ana is the elder sister,

(ii) their ages added together give 20 years,

(iii) the difference between their ages is 2 years.

Let x = Ana's age, in years and y = Tala's age, in years.

$x + y = 20$

$x - y = 2$

This is an example of a pair of simultaneous equations.

Example 1

Use a graph to solve the simultaneous equations:

$$x + y = 20$$
$$x - y = 2$$

Solution

We can rewrite the first equation to make y the subject.

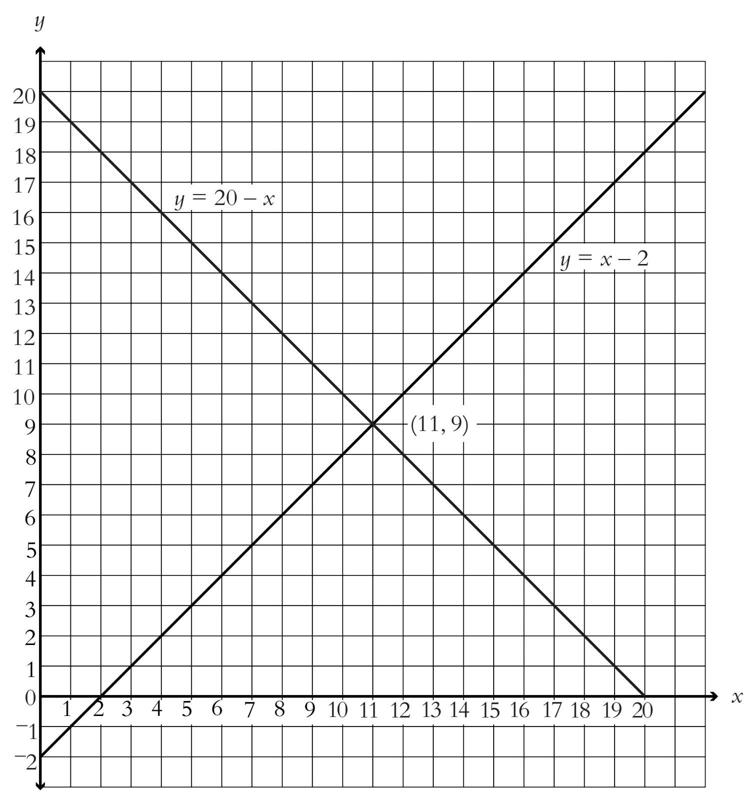
$$x + y = 20$$
$$y = 20 - x$$

For the second equation,

$$x - y = 2$$
$$x = y + 2$$
$$x - 2 = y$$

or $\quad y = x - 2$

Now draw the graphs $y = 20 - x$ and $y = x - 2$.



The lines cross at the point with co-ordinates (11, 9), so the solution of the pair of simultaneous equation is $x = 11$, $y = 9$.

Note: This means that the solution to the problem presented at the beginning is that Ana is aged 11 and Tala is aged 9.

Example 2

Use a graph to solve the simultaneous equations:

$x + 2y = 18$

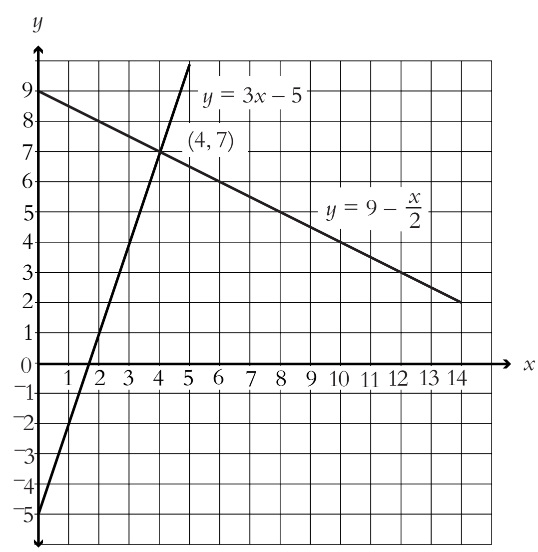
$3x - y = 5$

Solution

First rearrange the equations in the form $y = \ldots$

$$\begin{aligned} x + 2y &= 18 \\ 2y &= 18 - x \\ y &= \frac{18 - x}{2} \\ y &= 9 - \frac{x}{2} \end{aligned} \qquad\qquad \begin{aligned} 3x - y &= 5 \\ 3x &= y + 5 \\ 3x - 5 &= y \\ &\text{or} \\ y &= 3x - 5 \end{aligned}$$

Now draw these two graphs:



The lines cross at the point with co-ordinates (4, 7), so the solution is $x = 4$, $y = 7$.

An alternative approach is so solve simultaneous equations algebraically, as shown in the following examples.

Solving Simultaneous Equations Using Algebra

Simultaneous equations can also be solved without drawing graphs. Two algebraic methods can be used. Both methods change the two simultaneous equations into one equation which is then solved.

Method 1	Substitution	Put one of the equations into the other one (substitute).
Method 2	Elimination	Subtract one equation from the other to remove (eliminate) the x term. The x terms must be the same before subtraction.

Example 1

Solve the simultaneous equations:

$$\begin{aligned} x + 2y &= 29 \quad (1) \\ x + y &= 18 \quad (2) \end{aligned}$$

Solution

Note that the equations have been numbered (1) and (2).

Method 1 Substitution

Start with equation (2)

$$\begin{aligned} x + y &= 18 \\ y &= 18 - x \end{aligned}$$

Now replace y in equation (1)

Using $y = 18 - x$

$$\begin{aligned} x + 2y &= 29 \\ x + 2(18 - x) &= 29 \\ x + 36 - 2x &= 29 \\ 36 - x &= 29 \\ 36 &= 29 + x \\ 36 - 29 &= x \\ x &= 7 \end{aligned}$$

Finally, using $y = 18 - x$ gives

$$\begin{aligned} y &= 18 - 7 \\ y &= 11 \end{aligned}$$

So the solution is $x = 7$, $y = 11$.

Method 2 Elimination

The x terms are the same.

Take equation (2) away from equation (1).

$$\begin{aligned} x + 2y &= 29 \quad (1) \\ x + y &= 18 \quad (2) \\ \hline y &= 11 \quad (1) - (2) \end{aligned}$$

In equation (2), replace y with 11.

$$\begin{aligned} x + y &= 18 \\ x + 11 &= 18 \\ x &= 18 - 11 \\ x &= 7 \end{aligned}$$

Example 2

Solve the simultaneous equations:

$$2x + 3y = 28 \quad (1)$$

$$x + y = 11 \quad (2)$$

Solution

Method 1 Substitution

From equation (2)

$$\begin{aligned} x + y &= 11 \\ y &= 11 - x \end{aligned}$$

Substitute this into equation (1)

$$\begin{aligned} 2x + 3(11 - x) &= 28 \\ 2x + 33 - 3x &= 28 \\ 33 - x &= 28 \\ 33 &= 28 + x \\ 33 - 28 &= x \\ x &= 5 \end{aligned}$$

Finally use

$$\begin{aligned} y &= 11 - x \\ y &= 11 - 5 \\ y &= 6 \end{aligned}$$

So the solution is,

$x = 5$, $y = 6$

Method 2 Elimination

The x terms are not the same.

Multiply equation (2) by 2.

$$\begin{array}{rcll} 2x + 3y &=& 28 & (1) \\ \underline{2x + 2y} &=& \underline{22} & 2 \times (2) \\ y &=& 6 & (1) - 2 \times (2) \end{array}$$

Now replace y in equation (2) with 6.

$$\begin{aligned} x + 6 &= 11 \\ x &= 11 - 6 \\ x &= 5 \end{aligned}$$

So the solution is,

$x = 5$, $y = 6$

Example 3

Solve the simultaneous equations:

$$x - 2y = 8 \quad (1)$$

$$2x + y = 21 \quad (2)$$

Solution

Method 1 Substitution

From equation (2)

$$2x + y = 21$$
$$y = 21 - 2x$$

Substitute this into equation (1)

$$x - 2y = 8$$
$$x - 2(21 - 2x) = 8$$
$$x - 42 + 4x = 8$$
$$5x - 42 = 8$$
$$5x = 8 + 42$$
$$5x = 50$$
$$x = 10$$

Now substitute this into $y = 21 - 2x$

$$y = 21 - 2 \times 10$$
$$y = 21 - 20$$
$$y = 1$$

So the solution is,

$x = 10$, $y = 1$

Method 2 Elimination

The x terms are not the same.

Multiply equation (1) by 2.

$$\begin{array}{rcll} 2x + y & = & 21 & (2) \\ 2x - 4y & = & 16 & 2 \times (1) \\ \hline 5y & = & 5 & (2) - 2 \times (1) \\ y & = & 1 & \end{array}$$

Now replace this in equation (1).

$$x - 2y = 8$$
$$x = 8 + 2$$
$$x = 10$$

So the solution is,

$x = 10$, $y = 1$

Skill Exercises: Solving Simultaneous Equations

1. (a) Draw the lines with equations $y = 10 - x$ and $y = x + 2$.

   (b) Write down the co-ordinates of the point where the two lines cross.

   (c) What is the solution of the pair of simultaneous equations:

   $y = 10 - x$
   $y = x + 2$

2. (a) Draw the lines with equations $y = 5 - 2x$ and $y = 4 - x$.

   (b) Determine the co-ordinates of the point where the two lines cross.

   (c) Determine the solution of the simultaneous equations:

   $2x + y = 5$
   $x + y = 4$

3. Use a graphical method to solve the simultaneous equations:

   $x - 2y = 5$
   $x + y = 8$

4. Use a graph to solve the simultaneous equations:

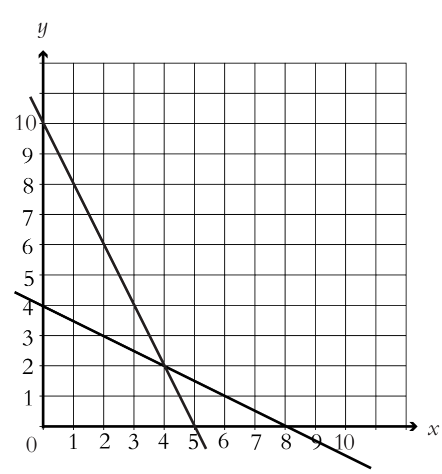
   $x + 2y = 10$
   $2x + 3y = 18$

5. Two numbers, x and y, are such that their sum is 24 and their difference is 6.

   (a) If the numbers are x and y, write down a pair of simultaneous equations in x and y.

   (b) Use a graph to solve the simultaneous equations and identify the two numbers.

6. Mele obtains the solution $x = 4, y = 2$ to a pair of simultaneous equations by drawing the following graph. What are the equations that she has solved?



7. A pair of simultaneous equations are given below:

   $2x + 4y = 14 \qquad (1)$

   $2x + y = 8 \qquad (2)$

   (a) Explain why subtracting equation (2) from equation (1) helps to solve the equations.

   (b) Solve the equations.

8. Solve the following pairs of simultaneous equations, using algebraic methods:

   (a) $x + 5y = 8$
   $x + 4y = 7$

   (b) $2x + 3y = 16$
   $8x + 3y = 46$

   (c) $2x + 6y = 26$
   $2x + 3y = 20$

   (d) $x + 2y = 3$
   $x + y = 7$

   (e) $x + 3y = 18$
   $x - 2y = 3$

   (f) $2x + 4y = 32$
   $2x - 3y = 11$

9. A pair of simultaneous equations is given below:

   $4x + 2y = 46 \qquad (1)$

   $x + 3y = 14 \qquad (2)$

   (a) Explain why you could calculate four times equation (2) – equation (1) to determine one solution.

   (b) Calculate the solution of this pair of equations.

10. Solve the following pairs of simultaneous equations, using an algebraic method:

   (a) $x + 2y = 7$
   $2x + 3y = 11$

   (b) $4x + 9y = 47$
   $x + 2y = 11$

   (c) $4x + 5y = 25$
   $x - y = 4$

   (d) $2x + 6y = 20$
   $x + 2y = 9$

   (e) $x - 8y = 4$
   $2x + y = 42$

   (f) $4x - 2y = 24$
   $8x - 3y = 50$

Simplifying Algebraic Expressions

When simplifying expression you should bring together terms that contain the same letter.

Note: x and x^2 must be treated as if they were different letters. You cannot add an x term to an x^2 term. The + and – signs go with the term which follows.

Example 1

Simplify each expression below:

(a) $4a + 3a + 6 + 2$ (b) $4a + 8b - 2a + 3b$

(c) $x^2 + 5x - 8x + x^2 - 4$ (d) $8x + y - 4x - 6y$

Solution

(a) The terms which involve a can be brought together. Also the 6 and 2 can be added:

$$4a + 3a + 6 + 2 = 7a + 8$$

(b) The terms involving a are considered together, and then the terms involving b:

$$\begin{aligned} 4a + 8b - 2a + 3b &= 4a - 2a + 8b + 3b \\ &= 2a + 11b \end{aligned}$$

(c) Here the x and the x^2 must be treated as if they are different letters:

$$\begin{aligned} x^2 + 5x - 8x + x^2 - 4 &= x^2 + x^2 + 5x - 8x - 4 \\ &= 2x^2 - 3x - 4 \end{aligned}$$

(d) The different letters, x and y, must be considered in turn:

$$\begin{aligned} 8x + y - 4x - 6y &= 8x - 4x + y - 6y \\ &= 4x - 5y \end{aligned}$$

When a bracket is to be multiplied by a number or a letter, every term inside the bracket must be multiplied.

Example 2

Remove the brackets from each expression below:

(a) $6(x + 5)$ (b) $3(2x + 7)$ (c) $4(x - 3)$ (d) $x(x - 4)$

Solution

(a) $6(x + 5) = 6 \times x + 6 \times 5 = 6x + 30$

(b) $3(2x + 7) = 3 \times 2x + 3 \times 7 = 6x + 21$

(c) $4(x - 3) = 4 \times x - 4 \times 3 = 4x - 12$

(d) $x(x - 4) = x \times x - x \times 4 = x^2 - 4x$

Skill Exercises: Simplifying Algebraic Expressions

1. Simplify each of these expressions:

(a) $a + 2a + 3a$
(b) $3a + 2 + 4 + 6$
(c) $3a + 2b + 8a + 4b$
(d) $4x + 2y + 8y + y$
(e) $5x + 2y + 8x - 3y$
(f) $6a + 7b + 3b - 4a$
(g) $4 + 6a - 3a + 2 + b$
(h) $p + q + 2p - 8q + 3p$
(i) $x + y - 8x + 2y$
(j) $4x - 3p + 2p - 2x$
(k) $7x - 4z + 8x - 5z$
(l) $3z - 4x + 2z - 10x$
(m) $3q - 4x + 8a - 2x + q$
(n) $x + y + z - p - q - y$
(o) $x + y + y + 4 + 2x - 3y$
(p) $4x - 8q + 17x - 24q$
(q) $^-x + y + x + y$
(r) $4x + 7y - 3x - 8y + x + y$
(s) $^-8x + 7y - 11x + 4y$
(t) $6x - 18y + 17x - 4$
(u) $x + y - 8x - 11y$
(v) $4p + 8q - 8p - 4q$

2. Simplify each of the following expressions:

(a) $2x^2 + 3x + 4x^2 + 5x$
(b) $x^2 + 8x + 5x + 10$
(c) $x^2 + 6x + 4x + x^2$
(d) $x^2 + x + 10 + x + 4x^2$
(e) $5x^2 - x - 6x^2 + 8x$
(f) $4x^2 - 3y^2 - x^2 + y^2$
(g) $x^2 + y^2 - x - y + x^2$
(h) $4x^2 - 7x + 1 + x^2 + 4x - 11$
(i) $x^2 - y^2 - x - y + 2x^2 - 2y^2$
(j) $y^2 + y - 4 + y + 4y^2$
(k) $ab + cd + 4ab$
(l) $xy + xz + xy + 4xz$
(m) $4ab + 7ab - 3ad$
(n) $4pq - 3qr + 5pq$

3. Remove the brackets from each expression below:

(a) $3(x + 5)$	(b) $4(6 + x)$	(c) $7(x + 2)$
(d) $2(x + 6)$	(e) $5(x + 2)$	(f) $4(2x + 3)$
(g) $5(3x + 2)$	(h) $8(5x + 3)$	(i) $7(x - 6)$
(j) $8(5 - x)$	(k) $4(2x - 7)$	(l) $7(5x - 3)$
(m) $6(3x - 5)$	(n) $4(x - 2y)$	(o) $5(x + 2y + 3z)$
(p) $x(5 + x)$	(q) $a(2 - a)$	(r) $4(b - 3)$
(s) $2x(x - 6)$	(t) $4x(2x + 3)$	(u) $3x(7 - 2x)$
(v) $8x(x - 5)$		

4. Simplify each of the following expressions by first removing all the brackets.

(a) $3(a + 2) + 4(a + 5)$	(b) $2(2x + 4) + 3(x + 5)$
(c) $5(x + 2) + 3(x + 4)$	(d) $x(x + 1) + x(x + 6)$
(e) $4(x + 1) + x(x + 5)$	(f) $2(a + b) + 5(2a + 3b)$

Solving Linear Equations 2

Most equations require a number of steps to solve them. These steps must be logical so that the new equation still balances. Whatever you do to one side of an equation you must do the same to the other side. The following examples illustrate these steps.

Example 1

Solve the following equations:

(a) $3x + 7 = 13$ (b) $5x - 8 = 13$

(c) $\dfrac{x}{5} = 3$ (d) $4(x - 3) = 8$

Solution

(a) First subtract 7 from both sides of the equation:

$$\begin{aligned} 3x + 7 &= 13 \\ 3x + 7 - 7 &= 13 - 7 \\ 3x &= 6 \end{aligned}$$

Next divide both sides of the equation by 3:

$$\begin{aligned} \frac{{}^{1}\cancel{3}x}{\cancel{3}_{1}} &= \frac{\cancel{6}^{2}}{\cancel{3}_{1}} \\ x &= 2 \end{aligned}$$

(b) First add 8 to both sides of the equation:

$$\begin{aligned} 5x - 8 &= 13 \\ 5x - 8 + 8 &= 13 + 8 \\ 5x &= 21 \end{aligned}$$

Then divide both sides of the equation by 5:

$$\begin{aligned} \frac{{}^{1}\cancel{5}x}{\cancel{5}_{1}} &= \frac{21}{5} \\ x &= \frac{21}{5} \\ x &= 4\frac{1}{5} \end{aligned}$$

(c) First add 2 to both sides of the equation:

$$\begin{aligned} \frac{x}{5} - 2 &= 3 \\ \frac{x}{5} - 2 + 2 &= 3 + 2 \\ \frac{x}{5} &= 5 \end{aligned}$$

Then multiply both sides of the equation by 5:

$$\begin{aligned} \frac{x}{\cancel{5}_{1}} \times \cancel{5}^{1} &= 5 \times 5 \\ x &= 25 \end{aligned}$$

(d) First remove the brackets, multiply each term inside the bracket by 4:

$$\begin{aligned} 4(x - 3) &= 8 \\ 4x - 12 &= 8 \end{aligned}$$

Then add 12 to both sides of the equation:

$$\begin{aligned} 4x - 12 + 12 &= 8 + 12 \\ 4x &= 20 \end{aligned}$$

Finally divide both sides by 4:

$$\begin{aligned} \frac{{}^{1}\cancel{4}x}{\cancel{4}_{1}} &= \frac{\cancel{40}^{5}}{\cancel{4}_{1}} \\ x &= 5 \end{aligned}$$

Sometimes equations may contain an x or a ^{-}x on both sides of the equation. The next examples show how to deal with these cases.

Example 2

Solve these equations:

(a) $4x + 6 = 3x + 10$ (b) $6 - 2x = 8$ (c) $4x - 2 = 8 - 6x$

Solution

(a) As x appears on both sides of the equation, first subtract $3x$ from both sides:

$$\begin{aligned} 4x + 6 &= 3x + 10 \\ 4x + 6 - 3x &= 3x + 10 - 3x \\ x + 6 &= 10 \end{aligned}$$

Then subtract 6 from both sides:

$$\begin{aligned} x + 6 - 6 &= 10 - 6 \\ x &= 4 \end{aligned}$$

(b) As the left-hand side contains $^{-}2x$, add $2x$ to both sides:

$$\begin{aligned} 6 - 2x &= 8 \\ 6 - 2x + 2x &= 8 + 2x \\ 6 &= 8 + 2x \end{aligned}$$

Then subtract 8 from both sides:

$$\begin{aligned} 6 - 8 &= 8 + 2x - 8 \\ ^{-}2 &= 2x \end{aligned}$$

Finally divide both sides by 2:

$$\begin{aligned} \frac{{}^{-1}\cancel{^{-}2}}{\cancel{2}_1} &= \frac{{}^{1}\cancel{2}x}{\cancel{2}_1} \\ ^{-}1 &= x \quad \text{or} \quad x = {}^{-}1 \end{aligned}$$

(c) As one side contains $^{-}6x$, add $6x$ to both sides:

$$\begin{aligned} 4x - 2 &= 8 - 6x \\ 4x - 2 + 6x &= 8 - 6x + 6x \\ 10x - 2 &= 8 \end{aligned}$$

Then add 2 to both sides of the equation:

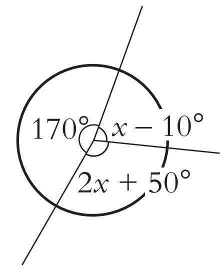
$$\begin{aligned} 10x - 2 + 2 &= 8 + 2 \\ 10x &= 10 \end{aligned}$$

Finally divide both sides by 10:

$$\begin{aligned} \frac{{}^{1}\cancel{10}x}{{}_{1}\cancel{10}} &= \frac{\cancel{10}^{1}}{\cancel{10}_1} \\ x &= 1 \end{aligned}$$

Example 3

Use the information in the diagram to write down an equation and then find the value of x.



Solution

The three angles shown must add up to 360°, so:

$$170 + 2x + 50 + x - 10 = 360$$
$$210 + 3x = 360$$

Subtracting 210 from both sides gives:

$$210 + 3x - 210 = 360 - 210$$
$$3x = 150$$

Then dividing both sides by 3 gives:

$$x = 50°$$

Skill Exercises: Solving Linear Equations 2

1. Solve each of these equations:

(a) $3x + 6 = 48$ (b) $5x - 6 = 39$ (c) $2x - 6 = 22$

(d) $6x - 7 = 41$ (e) $8x - 3 = 29$ (f) $6x + 12 = 20$

(g) $4x + 18 = 2$ (h) $5x + 10 = 5$ (i) $3x + 6 = 1$

(j) $5(x + 2) = 45$ (k) $3(x - 2) = 12$ (l) $2(x + 7) = 10$

(m) $3(2x - 1) = 57$ (n) $3(2x + 7) = 27$ (o) $5(5x + 1) = 20$

(p) $4(2x + 3) = {}^-8$ (q) $5(3x - 1) = {}^-2$ (r) $2(8x + 5) = {}^-2$

(s) $6x - 8 = {}^-26$ (t) $4(x + 15) = 60$ (u) $5x - 8 = {}^-10$

(v) $\frac{x}{4} - 1 = 8$ (w) $\frac{x}{3} + 2 = 7$ (x) $\frac{2x}{5} + 1 = 3$

2. Solve these equations:

(a) $2x + 6 = x + 3$ (b) $4x - 8 = 5x - 2$

(c) $6x + 7 = 2x + 20$ (d) $x + 6 = 2x - 8$

(e) $3x + 7 = 2x + 11$ (f) $10x + 2 = 8x + 22$

(g) $6 - x = 5$ (h) $2 - x = 5$

(i) $3 - x = {}^-10$ (j) $14 - 3x = 5$

(k) $10 - 2x = 2$ (l) $4 - 3x = 2$

(m) $x + 2 = 8 - x$ (n) $x + 4 = 10 - 2x$

(o) $x + 4 = 9 - 2x$ (p) $8 - x = 12 - 2x$

(q) $22 - 4x = 18 - 2x$ (r) $3 - 6x = 2 - 4x$

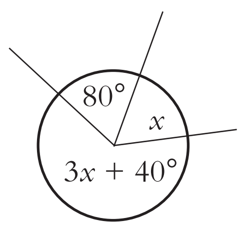
(s) $3(x + 2) = 5(x - 2)$ (t) $4 = 8 - \frac{x}{3}$

(u) $3 - \frac{x}{4} = {}^{-}5$ (v) $4(x - 2) = 3(x + 2)$

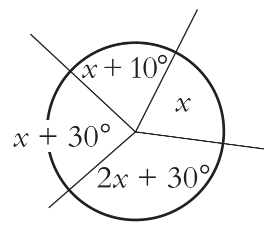
(w) $5 = 18 - \frac{x}{3}$ (x) $2 - \frac{x}{4} = 1 - \frac{x}{6}$

3. For each diagram below, write down an equation involving x and then solve it.

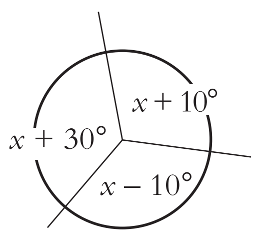
(a)



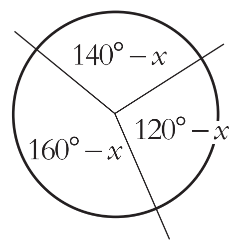
(b)



(c)



(d)



4. A rope of length 10 m is used to mark out a rectangle so that the two long sides are 1 m longer than the short sides. If x is the length of the short sides, write down an equation to describe this situation and hence find x.

5. You ask a friend to think of a number, double it and add 10. His answer is 42. If x is the number your friend thought of, write down the relevant equation and find x.

6. Six teams enter a competition. There are x members in each team. If eight people drop out and 34 complete the competition, write down an equation and solve it to find the number in each team at the start of the competition.

7. Three people drive a car on a long journey. Sione drives for two hours more than Mele. Filipo drives for twice as long as Mele. The whole journey takes six hours. Use an equation to find out how long each person drives.

8. A driver travels 80 kilometres to a motorway and then travels at a steady 60 km/h for x hours. Write down and solve equations involving x if the driver travels a total of:

   (a) 290 kilometres (b) 220 kilometres

   Give your answers in hours and minutes.

9. A student was asked to think of a number and follow these instructions. In each case, let x be the number the student thinks of. Write down an equation, and find the value of x.

   (a) Think of a number, add 6 and double it. Answer 18

   (b) Think of a number, divide by 2 and add 10. Answer 16

   (c) Think of a number, divide by 2, add 2 and multiply by 2. Answer 9

   (d) Think of a number, subtract 7, divide by 2 and multiply by 10. Answer 115

10. Four consecutive numbers, when added together, give a total of 114. If x is the lowest number, write down an equation and solve it.

11. Ati thinks of a number, he doubles it and then adds five. The answer is 17. What was his number?

12. (a) Write, in symbols, the rule:

    To find y, double x and add 1

    (b) Use your rule from part (a) to calculate the value of x when $y = 9$.

13. Keleti uses this rule:

    Start with a number, divide it by 2 and then add 3. Write down the result.

    (a) What is the result when Keleti starts with 8?

    (b) What number did Keleti start with when the result is 5?

14. Solve the equation:

    $11x + 5 = x + 25$

15. Solve the equations:

    (a) $4x - 1 = 17$ (b) $11y + 3 = 5y + 27$

Solving Quadratic Equations by Factorisation

Equations of the form:

$$\boxed{ax^2 + bx + c = 0}$$

are called quadratic equations. Many can be solved using factorisation. If a quadratic equation can be written as:

$(x - a)(x - b) = 0$

then the equation will be satisfied if either bracket is equal to zero. That is:

$(x - a) = 0$ or $(x - b) = 0$

$\therefore$ there would be two possible solutions, $x = a$ and $x = b$.

Example 1

Solve $x^2 + 6x + 5 = 0$

Solution

Factorising gives:

$$\begin{array}{lrcllrcl} & (x+5)(x+1) & = & 0 & & & & \\ \therefore & x+5 & = & 0 & \text{or} & x+1 & = & 0 \\ \text{therefore,} & x & = & {}^-5 & \text{or} & x & = & {}^-1 \end{array}$$

Example 2

Solve $x^2 + 5x - 14 = 0$

Solution

Factorising gives:

$$\begin{array}{lrcllrcl} & (x-2)(x+7) & = & 0 & & & & \\ \therefore & x-2 & = & 0 & \text{or} & x+7 & = & 0 \\ \text{therefore,} & x & = & 2 & \text{or} & x & = & {}^-7 \end{array}$$

Example 3

Solve $x^2 - 12x = 0$

Solution

Factorising gives:

$$\begin{array}{lrcllrcl} & x(x-12) & = & 0 & & & & \\ \therefore & x & = & 0 & \text{or} & x-12 & = & 0 \\ \text{therefore,} & x & = & 0 & \text{or} & x & = & 12 \end{array}$$

Example 4

Solve $4x^2 - 81 = 0$

Solution

Factorising gives:

$$
\begin{array}{lrcl}
 & (2x-9)(2x+9) & = & 0 \\
\therefore & 2x-9 & = & 0 \quad \text{or} \quad 2x+9 = 0 \\
\text{therefore,} & x & = & -\frac{9}{2} \quad \text{or} \quad x = -\frac{9}{2} \\
 & & = & 4\frac{1}{2} \qquad\quad = {}^{-}4\frac{1}{2}
\end{array}
$$

Example 5

Solve $x^2 - 4x + 4 = 0$

Solution

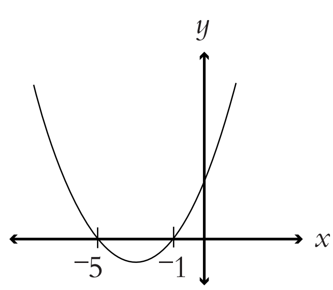
Factorising gives:

$$
\begin{array}{lrcl}
 & (x-2)(x-2) & = & 0 \\
\therefore & x-2 & = & 0 \quad \text{or} \quad x-2 = 0 \\
\text{therefore,} & x & = & 2 \quad \text{or} \quad x = 2
\end{array}
$$

This type of solution is often called a repeated solution.

Most of these examples have had two solutions, but the last example had only one solution. The graphs below show:

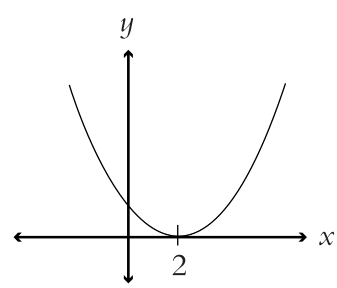
$y = x^2 + 6x + 5$



The curve crosses the x-axis at $x = {}^{-}5$ and $x = {}^{-}1$

These are the two solutions of: $x^2 + 6x + 5 = 0$

$y = x^2 - 4x + 4$



The curve touches the x-axis at $x = 2$

This is the one solution of: $x^2 - 4x + 5 = 0$

Skill Exercises: Solving Quadratic Equations

1. Solve the following quadratic equations:

(a) $x^2 + x - 12 = 0$
(b) $x^2 - 2x - 15 = 0$
(c) $x^2 + 4x - 12 = 0$
(d) $x^2 + 6x = 0$
(e) $3x^2 - 4x = 0$
(f) $4x^2 - 9x = 0$
(g) $x^2 - 9 = 0$
(h) $x^2 - 49 = 0$
(i) $9x^2 - 64 = 0$
(j) $x^2 - 8x + 16 = 0$
(k) $x^2 + 10x + 25 = 0$
(l) $x^2 - 3x - 18 = 0$
(m) $x^2 - 11x + 28 = 0$
(n) $x^2 + x - 30 = 0$
(o) $x^2 - 14x + 40 = 0$
(p) $2x^2 + 7x + 3 = 0$
(q) $2x^2 + 5x - 12 = 0$
(r) $3x^2 - 7x + 4 = 0$
(s) $4x^2 + x - 3 = 0$
(t) $2x^2 + 5x - 3 = 0$
(u) $2x^2 - 19x + 35 = 0$

2. The equations of a number of curves are given below. Find where each curve crosses the x-axis and use this to draw a sketch of the curve.

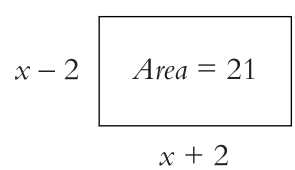
(a) $y = x^2 + 6x + 9$
(b) $y = x^2 - 4$
(c) $y = 2x^2 - 3x$
(d) $y = x^2 + x - 12$

3. Solve the following equations:

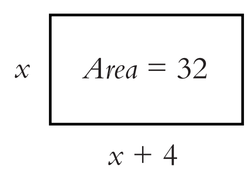
(a) $x^2 - 16 = 0$
(b) $x^2 - 625 = 0$
(c) $x^2 - 1 = 0$

4. Find the lengths of each side of the rectangles given below:

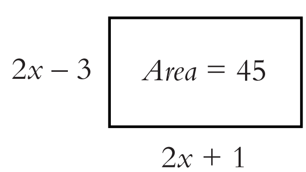
(a)



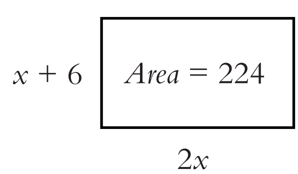
(b)



(b)



(d)



5. The height of a ball thrown straight up from the ground into the air at time, t, is given by:

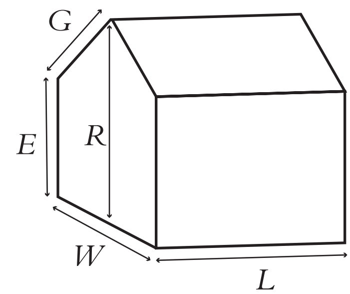
$h = 8t - 10t^2$

Find the time it takes for the ball to go up and fall back to ground level.

6. The diagram represents a greenhouse.

   The volume of the greenhouse is given by the formula:

   $V = \frac{1}{2}LW(E + R)$



   (a) Make L the subject of the formula, giving your answer as simply as possible.

   The surface area, A, of the greenhouse, is given by the formula:

   $$A = 2GL + 2EL + W(E + R),$$

   where $V = 500$, $A = 300$, $E = 6$ and $G = 4$.

   (b) By substituting these values into he equations for V and A show that L satisfies the equation:

   $$L^2 - 15L + 50 = 0$$

   Make the steps in your working clear.

   (c) Solve the equation $L^2 - 15L + 50 = 0$.

Plotting Quadratic Functions

The general formula for a quadratic graph is:

$$\boxed{y = ax^2 + bx + c}$$

where a, b and c are constants. In this section we investigate how changing the values of a, b and c changes the graph of the function.

Example 1

(a) Draw the graph $y = x^2$

(b) Draw the graph $y = x^2 - 1$

(c) Sketch the graph $y = x^2 + 1$, describing how it relates to the graph $y = x^2$.

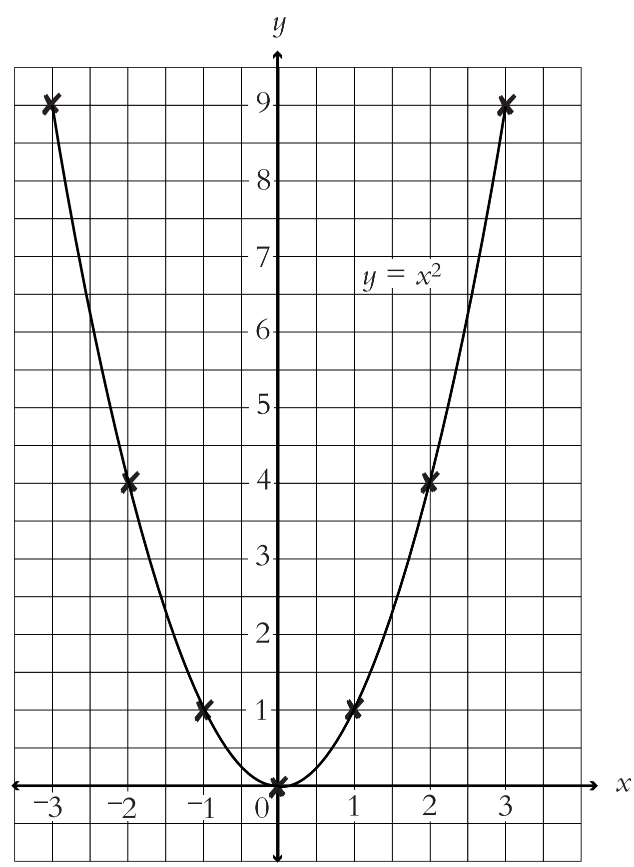
Solution

(a) The following table gives a set of values that can be used to draw a graph:

x	-3	-2	-1	0	1	2	3
x^2	9	4	1	0	1	4	9

The graph is plotted opposite.

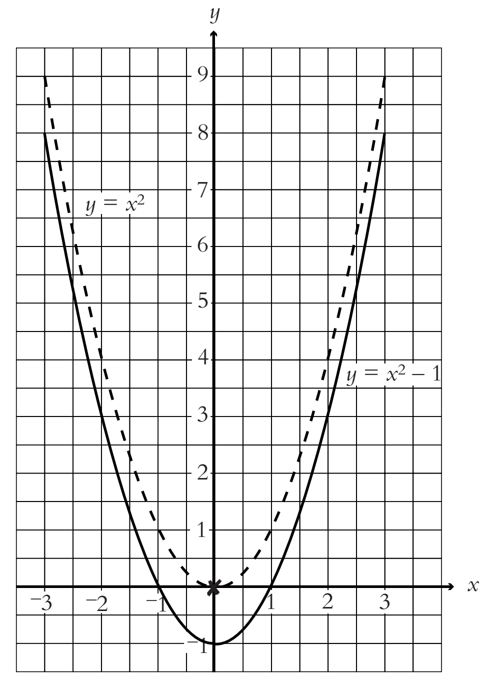
The shape formed is called a parabola.



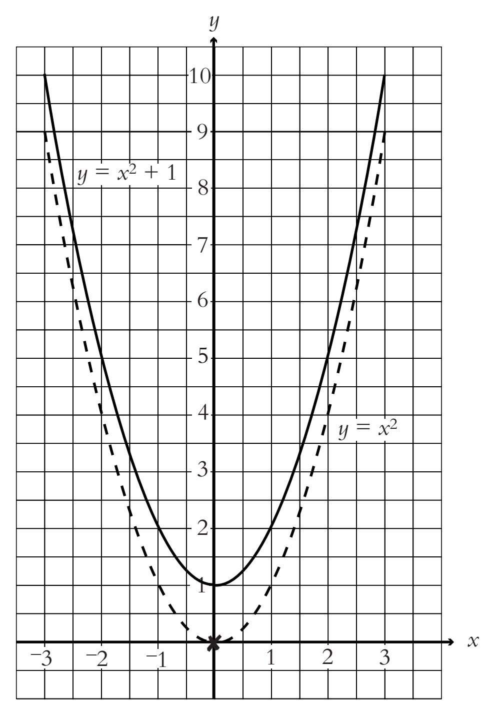
(b) This table gives values for $y = x^2 - 1$:

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	8	3	0	$^-1$	0	3	8

Note that the graph $y = x^2$ is translated downwards 1 unit to give the graph $y = x^2 - 1$.



(c) To get the graph $y = x^2 + 1$ the graph $y = x^2$ must be translated upwards by 1 unit, as shown in this diagram.



Example 2

(a) On the same set of axes, draw the graphs with equations:

$$y = x^2 \text{ and } y = \frac{1}{2}x^2$$

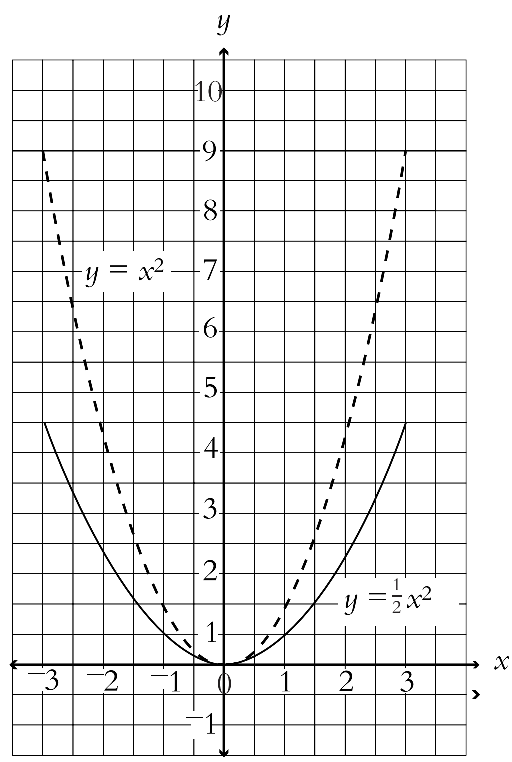
(b) Describe how the two graphs are related.

(c) Sketch the graphs $y = \frac{1}{4}x^2$ and $y = 2x^2$

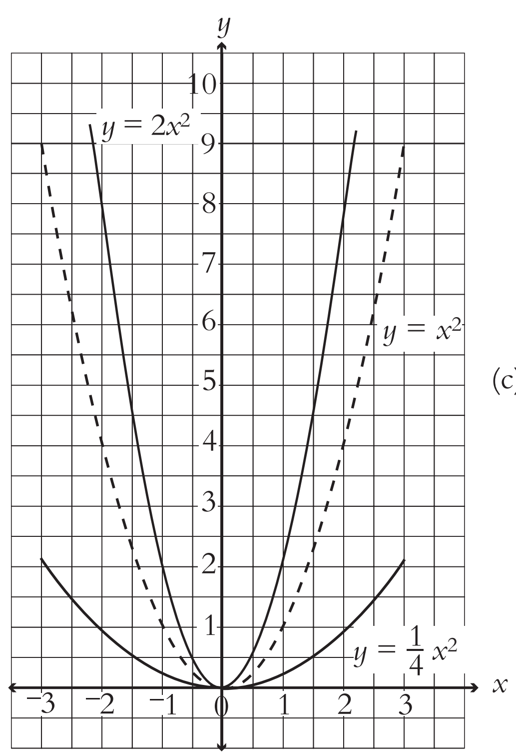
Solution

(a) The following table gives the values needed to plot the two graphs:

x	$^-3$	$^-2$	$^-1$	0	1	2	3
x^2	9	4	1	0	1	4	9
$\frac{1}{2}x^2$	4.5	2	0.5	0	0.5	2	4.5



(b) The graph $y = \frac{1}{2}x^2$ always has exactly half the height of the graph $y = x^2$, as shown opposite.



(c) The graph $y = \frac{1}{4}x^2$ is wider than $y = x^2$. The graph $y = 2x^2$ will be twice as thin as the graph $y = x^2$. The two graphs are shown in the diagram on the left.

Skill Exercises: Plotting Quadratic Functions

In the following exercises you will explore further how the values of a, b and c change the shape of a quadratic graph.

1. (a) Draw the graph $y = x^2$
   (b) Draw the graph $y = x^2 + 3$ on the same axes.
   (c) Draw the graph $y = x^2 - 2$ on the same axes.
   (d) Describe how the three graphs are related.

2. (a) Draw the graph $y = {}^{-}x^2$
   (b) On the same set of axes, draw the graphs:
   (i) $y = 4 - x^2$ (ii) $y = 9 - x^2$
   (iii) $y = 1 - x^2$ (iv) $y = {}^{-}1 - x^2$

3. On the same set of axes, sketch the graphs:
   (a) $y = x^2$ (b) $y = 3x^2$
   (c) $y = 4x^2$ (d) $y = \frac{3}{4}x^2$

4. On the same set of axes, sketch the graphs:
   (a) $y = {}^{-}x^2$ (b) $y = \frac{-1}{2}x^2$
   (c) $y = \frac{-1}{4}x^2$ (d) $y = \frac{-3}{4}x^2$

5. (a) Plot the graphs with equations:
   $y = 2x^2$ and $y = x^2 + 4$
   (b) What are the co-ordinates of the points where the two curves intersect?

6. (a) Draw the graphs with equations:
   $y = (x + 1)^2$, $y = (x + 3)^2$ and $y = (x - 2)^2$
   (b) Describe how each graph is related to the graph, $y = x^2$
   (c) On a new set of axes, sketch the graphs with equations:
   $y = (x - 5)^2$, $y = (x - 3)^2$ and $y = (x + 4)^2$

7. Sketch the graphs with the following equations:

   (a) $y = (x + 1)^2 + 1$ (b) $y = (x - 2)^2 - 3$

   (c) $y = (x + 4)^2 - 3$ (d) $y = (x - 3)^2 + 2$

8. (a) Draw the graphs with equations:

   $y = x^2 + x$ $y = x^2 + 2x$

   $y = x^2 + 4x$ $y = x^2 + 6x$

   (b) For each graph, write down the co-ordinates of the lowest point. What would be the co-ordinates of the lowest point of the curve $y = x^2 + bx$?

   (c) Draw the graphs of the curves with equations:

   $y = x^2 - x$, $y = x^2 - 4x$ and $y = x^2 - 6x$

   (d) What would be the co-ordinates of the lowest point of the curve with equation $y = x^2 - bx$?

9. (a) Draw the graphs with equations:

   $y = 2x^2 + 4x + 1$ and $y = 3x^2 + 6x + 2$

   (b) Where does each curve intersect the y-axis?

   (c) Where does the curve $y = ax^2 + bx + c$ intersect the y-axis?

   (d) Write down the co-ordinates of the lowest point of each of the curves drawn in part (a).

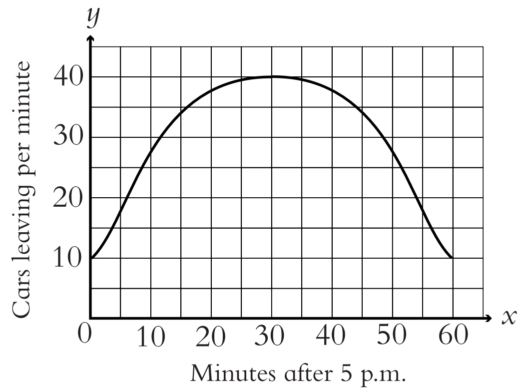
   (e) What are the co-ordinates of the lowest point of the curve $y = ax^2 + 2ax + c$?

10. (a) Plot the graphs:

    $y = x^2 + 5x + 1$, $y = 2x^2 + 8x - 1$ and $y = 3x^2 - 9x + 7$

    (b) What are the co-ordinates of the lowest point of the curve $y = ax^2 + bx + c$?

11. The graph shows the rate at which cars left a car park from 5 p.m. to 6 p.m.



The lowest rate was 10 cars per minute at 5 p.m. and 6 p.m. The highest rate was 40 cars per minute at 5:30 p.m.

$y = ax^2 + bx + c$ is the relationship between y, the number of cars leaving per minute, and x, the number of minutes after 5 p.m.

(a) Explain how you can work out from the graph that the value of c is 10.

(b) Use the graph to form equations to work out the values of a and b in the equation $y = ax^2 + bx + c$

Show your working.

Unit 4: MEASUREMENT

In this unit you will be:

4.1 Working with Surface Areas

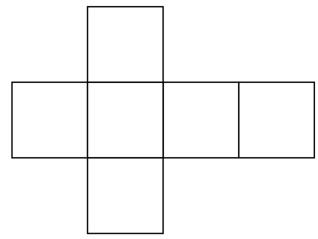
- Calculating Surface Areas of Solid Figures.

Section 4.1 Working With Surface Areas

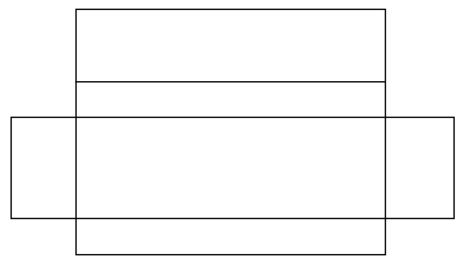
Calculating Surface Areas of Solid Figures

The net of a cube can be used to find its surface area.

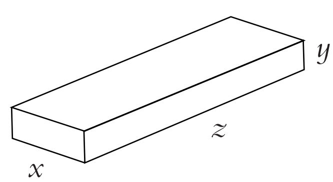
The net is made up of 6 squares, so the surface area will be 6 times the area of one square. If x is the length of the sides of the cube, its surface area will be $6x^2$.

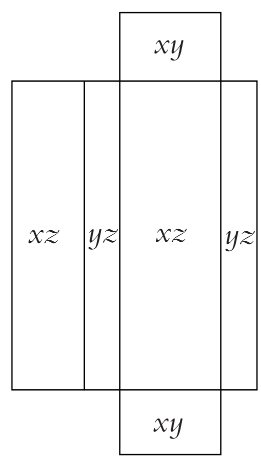


This diagram shows the net for a cuboid. To find the surface area, the area of each of the 6 rectangles must be found and then added to give the total.



If x, y and z are the lengths of the sides of the cuboid, then the area of the rectangles in the net are shown here:

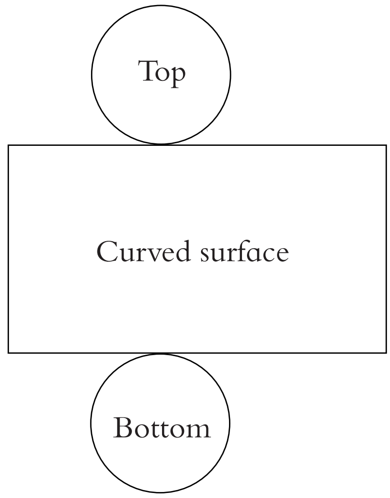




The total surface area of the cuboid is then given by:

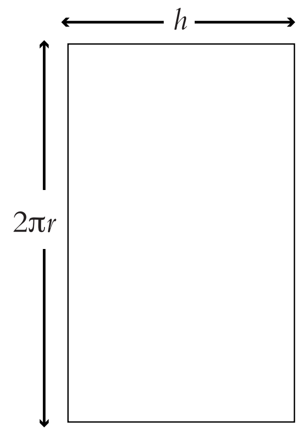
$$\boxed{A = 2xy + 2xz + 2yz}$$

To find the surface area of a cylinder, consider how a cylinder can be broken up into three parts, the top, bottom and curved surface.



The areas of the top and bottom are the same and each is given by πr^2

The curved surface is a rectangle. The length of one side is the same as the circumference of the circles, $2\pi r$, and the other side is simply the height of cylinder, h. So the area is $2\pi rh$

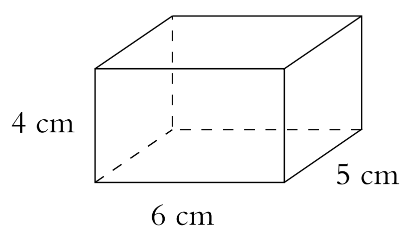


The total surface area of the cylinder is:

$$2\pi r^2 + 2\pi rh$$

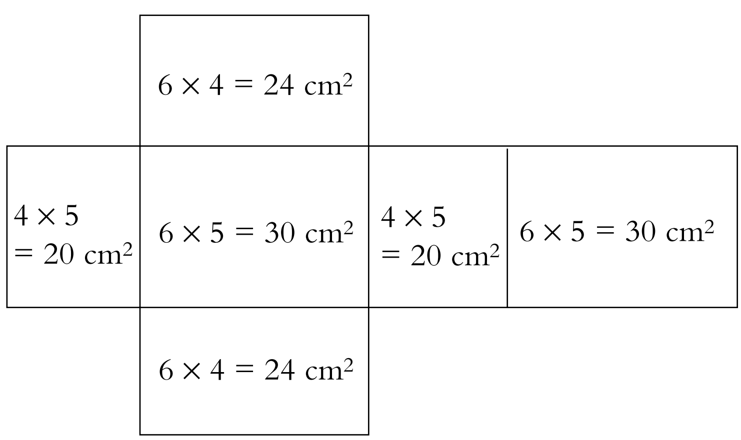
Example 1

Find the surface area of the cuboid shown in the diagram.



Solution

The diagram shows the net of the cuboid and the areas of the rectangles that it contains.



Using the net, the total surface area is given by:

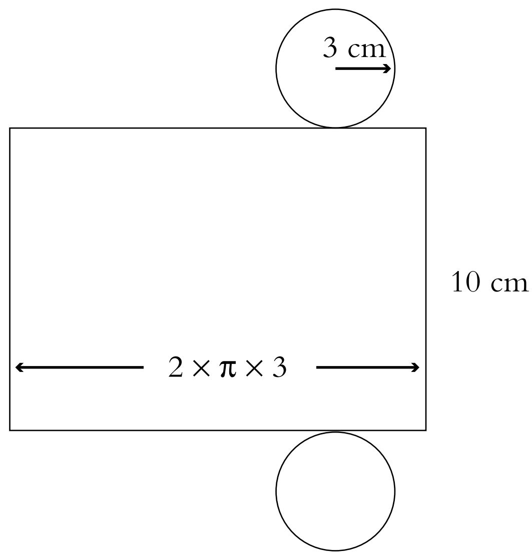
$$A = 2 \times 20 + 2 \times 30 + 2 \times 24$$
$$= 148 \text{ cm}^2$$

Example 2

Cans are made out of aluminium sheets and are cylinders of radius 3 cm and height 10 cm. Find the area of aluminium needed to make one can.

Solution

The diagram shows the two circles and the rectangle from which cans will be made.



The rectangle has one side as the height of the cylinder (10 cm) and the other side, being the circumference of the top and bottom, is $2 \times \pi \times 3$ cm.

The area of the rectangle is: $10 \times 2 \times \pi \times 3$

The area of each circle is: $\pi \times 3^2$

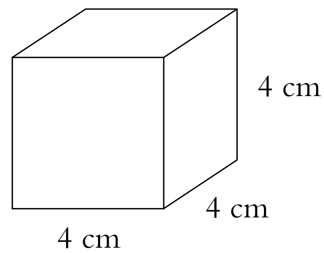
So the total surface area is:

$$A = 10 \times 2 \times \pi \times 3 + 2 \times \pi \times 3^2$$
$$= 245.04 \text{ cm}^2 \text{ (to 2 d.p.)}$$

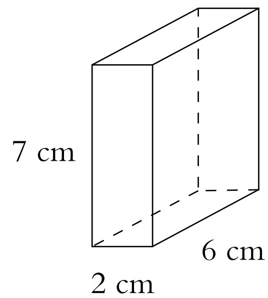
Skill Exercises: Calculating Surface Areas of Solid Figures

1. Find the surface area of each of the following cubes or cuboids:

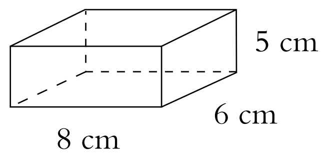
(a)



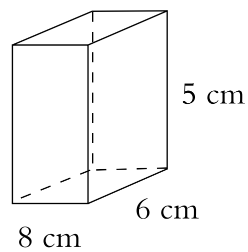
(b)



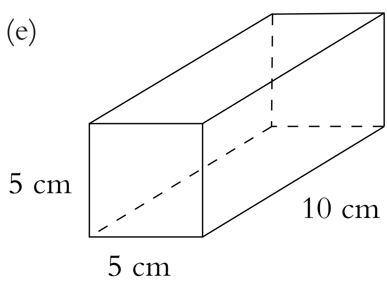
(c)



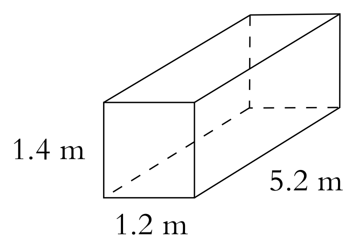
(d)



(e)

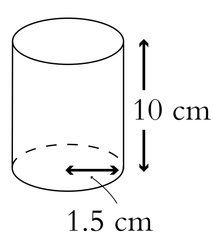


(f)

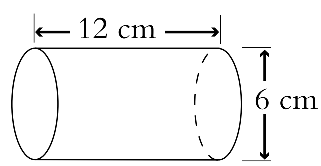


2. Find the total surface area of each cylinder shown below:

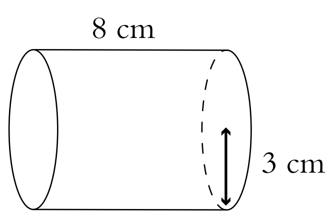
(a)



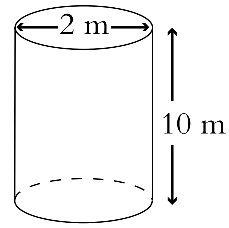
(b)



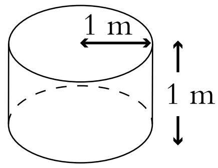
(c)



(d)



(e)



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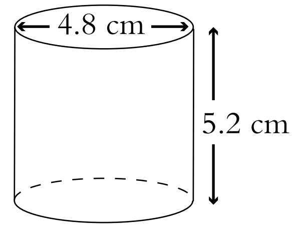
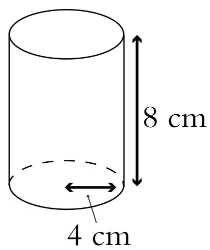
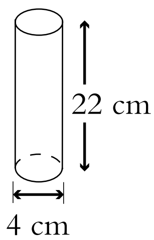


3. Show that each of the cylinders below has the same surface area and find which has the biggest volume.

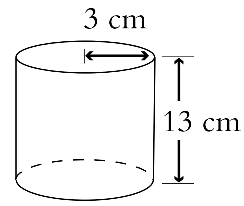
(a)



(b)

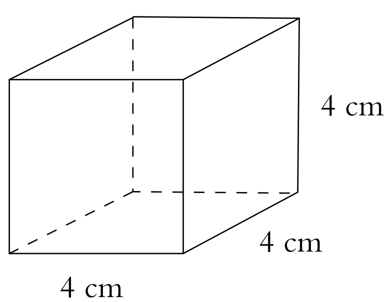


(c)

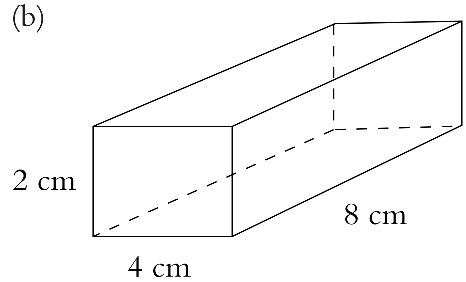


4. Show that each of the three cuboids below has the same volume. Which has the smallest surface area?

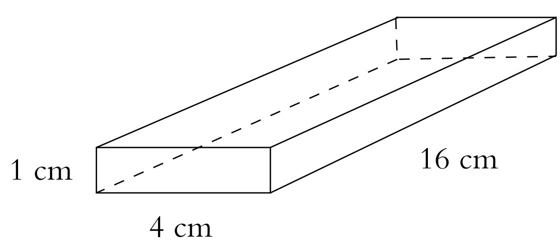
(a)



(b)



(c)

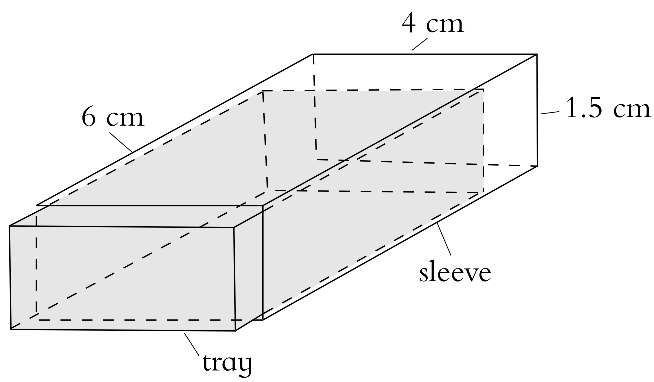


5. A gardener uses a roller to flatten the grass on a lawn. The roller consists of a cylinder of radius 30 cm and width 70 cm.
   (a) Find the area of grass that the roller covers as the cylinder completes 1 rotation.
   (b) If the roller is pulled 5 m, what area of grass does the roller flatten?

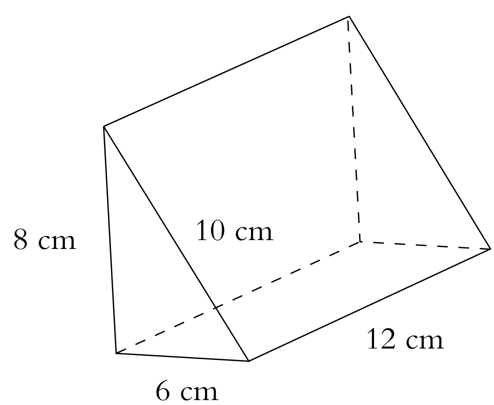
6. The volume of a cube is 343 cm^3. Find the surface area of the cube.

7. The surface area of a cube is 150 cm^2. Find the volume of the cube.

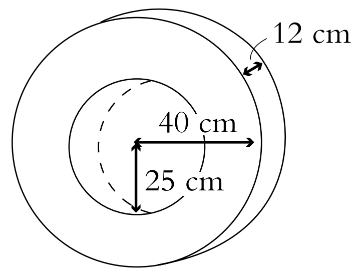
8. A matchbox consists of a tray that slides into a sleeve. If the tray and sleeve have the same dimensions and no material is used up in joins, find:
   (a) The area of card needed to make the tray;
   (b) The area of card needed to make the sleeve;
   (c) The total area of the card needed to make the matchbox.



9. Draw a net of the prism shown in the diagram and use it to find the surface area of the prism.



10. A car tyre can be thought of as a hollow cylinder with a hole cut out of the centre. Find the surface area of the outside of the tyre.



Unit 1: ANSWERS — NUMBER

Section 1.1 Working With Real Numbers

(Pg. 8) Skill Exercises: Recognising Real Numbers

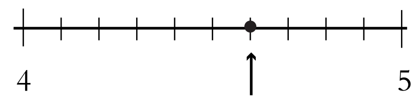
1. (a) Rational (b) Rational
   (c) Irrational (d) Rational (recurring)
   (e) Rational (terminating) (f) Irrational
   (g) Irrational (h) Rational (terminating)
   (i) Irrational (j) Rational (recurring).

2. (a) $\frac{49}{100}$ (b) $\frac{1}{3}$ (c) $\frac{7}{4}$
   (d) Irrational (e) $\frac{417}{1000}$ (f) $\frac{1}{10}$
   (g) $\frac{1}{11}$ (h) $\frac{6}{11}$ (i) $\frac{1}{8}$
   (j) $\frac{481}{500}$

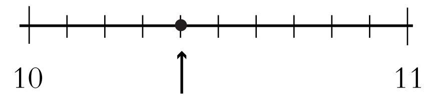
(Pg. 9) Skill Exercises: Graphing Real Numbers

1. (a) 5.6 (b) 3.3 (c) 7.8
   (d) 6.42 (e) 7.17 (f) 3.73
   (g) 4.6 (h) 4.8 (i) 3.16
   (j) 3.94 (k) 10.2 (l) 1.4

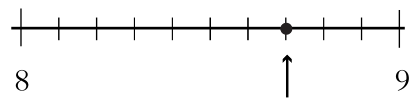
2. (a) 4.6



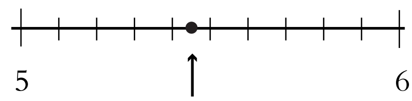
(b) 10.4



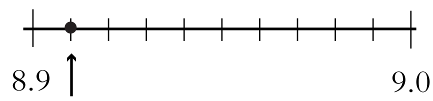
(c) 8.7



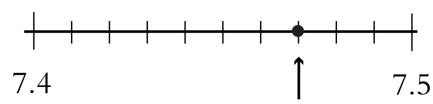
(d) 5.45



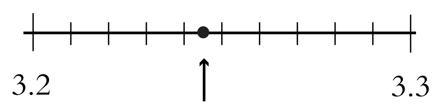
(e) 8.91



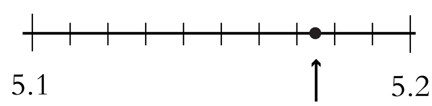
(f) 7.47



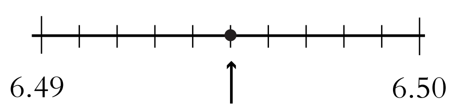
(g) 3.245



(h) 5.175



(i) 6.495



3. (a) hundredths (b) tenths
   (c) hundredths (d) tenths
   (e) thousandths (f) thousandths

4. 1.87

5. 76 cm

6. 0.8 kg

Section 1.2 Working With Negative Exponents

(Pg. 13) Skill Exercises: Revising Index Notation

1. (a) 4^5 (b) 3^3 (c) 6^7 (d) 7^4
   (e) 18^4 (f) 19^2 (g) 4^6 (h) 7^5
   (i) 10^6 (j) 100^5

2. (a) 81 (b) 625 (c) 2401 (d) 10 000
   (e) 1 (f) 729 (g) 128 (h) 2
   (i) 4096 (j) 4 (k) 1 (l) 25

3. (a) 2^{11} (b) 3^9 (c) 3^{13} (d) 4^5
   (e) 5^4 (f) 5^5 (g) 4^2 (h) 5^3
   (i) 3^2 (j) 7^{14} (k) 17^2 (l) 9^4
   (m) 4^5 (n) 4^{16} (o) 3^6 (p) $3^0 = 1$
   (q) $3^1 = 3$ (r) 3^5 (s) 3^7 (t) 4^7
   (u) $5^0 = 1$

4. (a) 2^2 (b) 2^3 (c) 2^4 (d) 2^6
   (e) 3^3 (f) 5^2 (g) 4^3 (h) 3^4
   (i) 5^3

5. (a) 3^{13} (b) 2^8 (c) 4^{11} (d) 3^{10}
   (e) 2^9 (f) 2^{10} (g) 3^5 (h) 3^7
   (i) 3^5 (j) 8^{10} (k) 7^3 (l) 9^2
   (m) 2^4 or 4^2 (n) 2^3 (o) 2^3

6. (a) 2^3 (b) 10^3 (c) 2^4 (d) 3^3
   (e) 3^4 (f) 10^4 (g) 5^4 (h) 4^3
   (i) 6^4 (j) 2^0 (k) 6^2 (l) 5^0

7. (a) 2^6 (b) 3^4 (c) 6^6 (d) 5^6
   (e) 2^8 (f) 4^6 (g) 3^8 (h) 5^8
   (i) 3^6

8. (a) 2^8 (b) 2^4 (c) 3^{10} (d) 5^3
(e) $(10^5)^3$ (f) $(7^5)^4$

9. (a) 3^6 (b) 2^{14} (c) 5^{12} (d) 7^3
(e) 7^4 (f) 2^7 (g) $3^0 = 1$ (h) $4^1 = 4$
(i) $2^1 = 2$

10. (a) a^5 (b) a^{10} (c) x^9 (d) x^2
(e) y^3 (f) p^3 (g) q^3 (h) x^8
(i) b^3 (j) b^6 (k) c^3 (l) x^5
(m) y^2 (n) $x^0 = 1$ (o) x^8 (p) p^4
(q) x^3 (r) y^4 (s) $x^0 = 1$ (t) $x^1 = x$
(u) x^{12} (v) x^8 (w) x^{15} (x) x^{54}

(Pg. 16) Skill Exercises: Negative or Fractional Index Notation

1. (a) $\frac{1}{16}$ (b) $\frac{1}{8}$ (c) $\frac{1}{6}$ (d) $\frac{1}{7}$
(e) 3 (f) 8 (g) 2 (h) 3
(i) 1 (j) $\frac{1}{25}$ (k) 8 (l) 32
(m) 2187 (n) 125 (o) $\frac{1}{2}$

2. (a) -4 (b) -1 (c) -3 (d) $\frac{1}{2}$
(e) $\frac{-1}{2}$ (f) 2 (g) 3 (h) -1
(i) $\frac{3}{4}$ (j) -1 (k) -2 (l) -2
(m) -3 (n) $\frac{1}{2}$ (o) -2 (p) $\frac{3}{4}$
(q) $\frac{2}{3}$ (r) $\frac{2}{5}$

3. (a) 0.125 (b) 0.05 (c) 2 (d) 4

   (e) $\frac{1}{225} = 0.00444\ldots$ (f) 0.000125 (g) 729

   (h) 27 (i) $\frac{1}{2}$ (j) 1728 (k) 62748517

   (l) 1331

4. (a) $\frac{1}{a}$ (b) a^{10} (c) a^4 (d) $\frac{1}{a^6}$

   (e) $\frac{1}{a^2}$ (f) $\frac{1}{a^6}$ (g) a^8 (h) $a^{\frac{5}{2}}$

   (i) $\frac{1}{a^{\frac{3}{2}}}$ (j) a^2 (k) $\frac{1}{a^3}$ (l) a^3

   (m) $\frac{a^2}{b^2}$ (n) $\frac{a^6}{b^{12}}$ (o) $a^{12}b^2$ (p) $\frac{b^4}{a^4}$

   (q) $\frac{a^8}{b^{12}}$ (r) $\frac{m^2}{n^6}$ (s) $\frac{a^3}{b^5}$ (t) $\frac{m^2}{a}$

   (u) $\frac{c^3}{a^4b}$ (v) $\frac{x}{m^2}$ (w) $\frac{z^{12}}{x^8y^4}$ (x) $\frac{b^{\frac{16}{3}}}{a^2}$

5. (a) $\frac{1}{9}$ (b) a^4 (c) $y = 8$

Section 1.3 Working With Absolute Values

(Pg. 18) Skill Exercises: Calculating Absolute Values

1. (a) 7	(b) 16	(c) 24	(d) 1
(e) 3	(f) 6	(g) 13	(h) 2
(i) 21	(j) $^-5$	(k) $^-1$	(l) 46
(m) 45	(n) 5	(o) 14	

Unit 2: ANSWERS — ALGEBRA — PART 1

Section 2.1 Working With Algebraic Expressions

(Pg. 21) Skill Exercises: Substituting into Formulae

1. (a) 50	(b) 68	(c) 14	(d) 23
(e) $^{-}4$	(f) 59		
2. (a) 10	(b) 40	(c) 11.25	(d) 4
(e) $^{-}10$	(f) 7.04		
3. (a) 19.6	(b) 18.4	(c) 18.08	(d) 18.8
4. (a) $^{-}280$	(b) $^{-}40$	(c) 80	(d) 800
5. (a) 80	(b) 51	(c) ± 4	(d) ± 3
(e) $^{-}3$	(f) ± 5	(g) 0	(h) $\frac{3}{4}$
(i) 1	(j) 10	(k) $^{-}2$	(l) $^{-}10$
(m) ± 5	(n) 0.18	(o) 0.38	(p) ± 5
(q) ± 8	(r) ± 15		
6. (a) 3.8	(b) 0.225	(c) 2.6	(d) 7.5
(e) 9.7	(f) 2.4	(g) 0.5	(h) 7.12
(i) 3.7			

7. $^{-}21.67$°C (2 d.p.)

8. $^{-}13$

9. (a) $-\frac{13}{8}$ (b) $-\frac{5}{8}$

(Pg. 25) Skill Exercises: Changing the Subject

1. (a) $x = \frac{y}{4}$ (b) $x = \frac{y-3}{2}$ (c) $x = \frac{y+8}{4}$

   (d) $x = 4y - 2$ (e) $x = 5y + 2$ (f) $x = y - a$

   (g) $x = ya + b$ (h) $x = \frac{y-c}{a}$ (i) $x = \frac{yc-b}{a}$

   (j) $x = \frac{yb+c}{a}$ (k) $x = y - a - b$ (l) $x = yc + a - b$

   (m) $x = \frac{y}{ab}$ (n) $x = \frac{y-c}{ab}$ (o) $x = \frac{3cy+b}{4a}$

   (p) $x = \frac{pd+bc}{a}$ (q) $x = \frac{y}{b} - a$ (r) $x = \frac{4y}{a} - 3$

   (s) $x = \frac{2q}{3} + 4$ (t) $x = \frac{4v}{5} - y$ (u) $x = 4(z-a) + 3$

2. $I = \frac{V}{R}$; $R = \frac{V}{I}$

3. $m = \frac{F}{a}$; $a = \frac{F}{m}$

4. $r = \frac{C}{2\pi}$

5. (a) $t = \frac{v-u}{a}$ (b) $a = \frac{v-u}{t}$

6. $z = 3m - x - y$

7. (a) $a = \frac{v^2-u^2}{2s}$

8. $z = \frac{v}{xy}$

9. (a) $r = +\sqrt{\frac{V}{\pi h}}$ (only a positive value because r is radius)

   (b) 2.82

10. (a) $h = \frac{V}{x^2}$; $h = \frac{A-2x^2}{4x}$

    (b) 2

    (c) 2.5

(Pg. 28) Skill Exercises: Addition and Subtraction

1. (a) $\frac{9x}{20}$ (b) $\frac{11x}{28}$ (c) $\frac{8x}{15}$

(d) $\frac{41y}{21}$ (e) $\frac{23y}{20}$ (f) $\frac{13y}{7}$

(g) $\frac{19x}{70}$ (h) $\frac{x}{6}$ (i) $\frac{9x}{8}$

(j) $\frac{9x}{8}$ (k) $\frac{5a + 4b}{20}$ (l) $\frac{8x + 3y}{24}$

(m) $\frac{5a - 3b}{15}$ (n) $\frac{10a + 12b}{15}$ (o) $\frac{32a - 27b}{36}$

2. (a) $\frac{4y + 2x}{xy}$ (b) $\frac{6y - x}{xy}$ (c) $\frac{y + 3x}{xy}$

(d) $\frac{5}{a}$ (e) $\frac{8b + 3a}{2ab}$ (f) $\frac{10b - 3a}{6ab}$

(g) $\frac{25b + 12a}{15ab}$ (h) $\frac{23}{15a}$ (i) $\frac{17}{28a}$

(j) $\frac{21b - 16a}{24ab}$ (k) $\frac{1}{12a}$ (l) $\frac{11}{8a}$

3. (a) $\frac{2x + 1}{x(x + 1)}$ (b) $\frac{3x + 4}{x(x + 2)}$ (c) $\frac{7x + 3}{x(x + 1)}$

(d) $\frac{4x + 10}{x(x + 2)}$ (e) $\frac{4x + 2}{x(x + 2)}$ (f) $\frac{14x - 12}{3x(x + 3)}$

(g) $\frac{{}^{-}6}{x(x + 1)}$ (h) $\frac{6x - 10}{x(x - 5)}$ (i) $\frac{27x + 42}{5x(x + 6)}$

(j) $\frac{17x - 49}{2x(x - 7)}$ (k) $\frac{23x - 50}{3x(x - 10)}$ (l) $\frac{7x - 8}{3x(x - 8)}$

(Pg. 30) Skill Exercises: Multiplication and Division

1. (a) $\frac{a^2}{b^2}$ (b) $\frac{2a^3}{3b^2}$ (c) $\frac{12a^3}{b^3}$ (d) $\frac{4a^3}{3b^3}$

   (e) $\frac{x^3}{w^3}$ (f) $\frac{3x^3}{4ab}$ (g) $\frac{b^4}{4x^2}$ (h) $\frac{x^4}{9a}$

   (i) $\frac{5}{3ab^2}$

2. (a) 2 (b) $3a$ (c) 1 (d) $\frac{9ax}{16}$

   (e) $\frac{9ax}{16}$ (f) $3ab$ (g) $\frac{15}{d}$ (h) $\frac{7bc}{6}$

   (i) $2a^2$

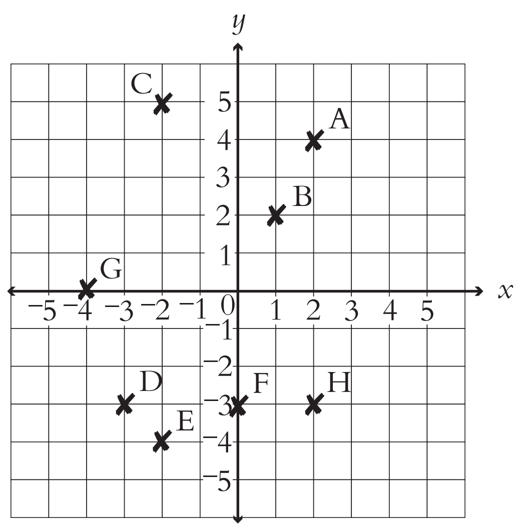
Unit 3: ANSWERS — ALGEBRA — PART 2

Section 3.1 Working With Graphs And Equations

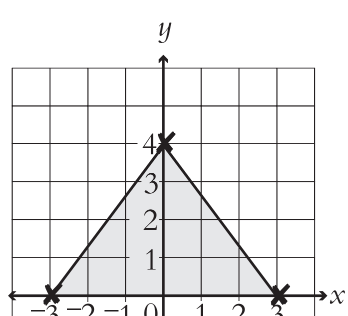
(Pg. 35) Skill Exercises: Plotting Co-ordinates

1. A(1, ⁻2) B(⁻4, 0) C(⁻2, ⁻3) D(3, 2)

   E(1, 4) F(0, 2) G(⁻2, 3) H(0, ⁻5)

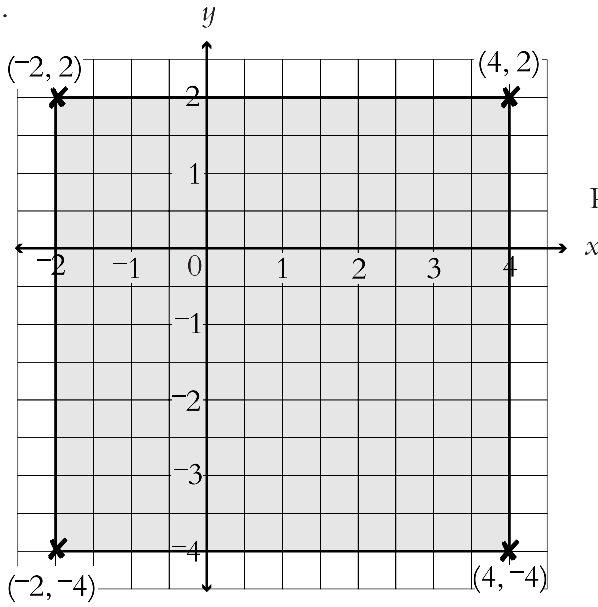
2. 

The points A, B and E all lie on a straight line through the origin.

3. 

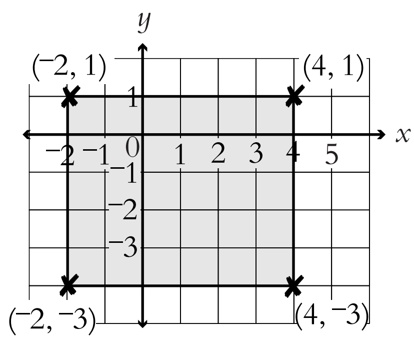
The shape is an isosceles triangle.

4.



Remaining corner (⁻2, ⁻4).

5.



Remaining corner (4, ⁻3).

6. (a) 3 units (b) (⁻1, ⁻2), (2, ⁻2) or (⁻1, 4), (2, ⁻4).

7. (a) (8, 8)

(b) (40, 40) because both co-ordinates are equal to double the tile number.

(c) Siaki is wrong because 25 is an odd number and all the corners with a ● have even numbers.

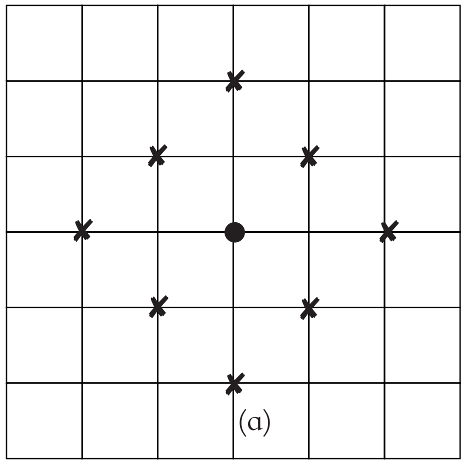
(d)

Tile Number	Co-ordinates of the Corner with a *x*
1	(2, 1)
2	(4, 3)
3	(6, 5)
4	(8, 7)

(e) Tile number 7 has a cross in the corner at (14, 13).

(f) Tile number 10 has a cross in the corner at (20, 19).

8. (a) and (b)



(c)

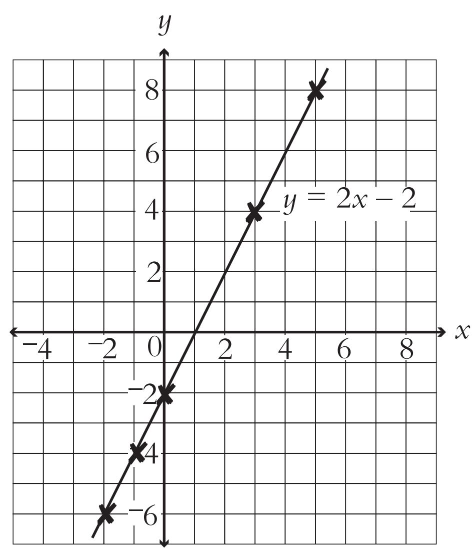
1st Step	S	W	W
2nd Step	W	S	W
3rd Step	W	W	S

(Pg. 43) Skill Exercises: Drawing Straight Line Graphs

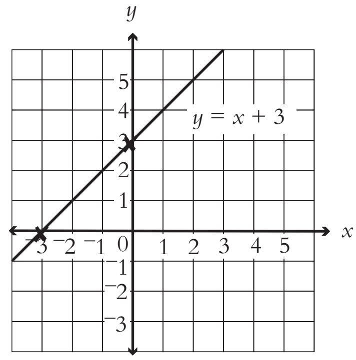
1. (a)

x	$^-2$	$^-1$	0	1	3	5
y	$^-6$	$^-4$	$^-2$	0	4	8

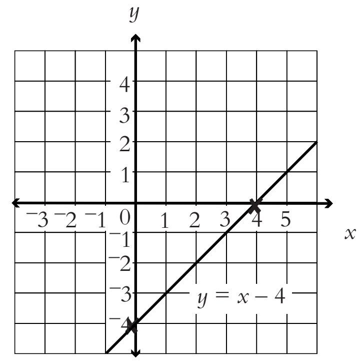
(b)



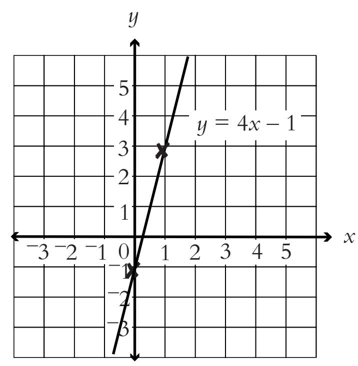
2. (a)



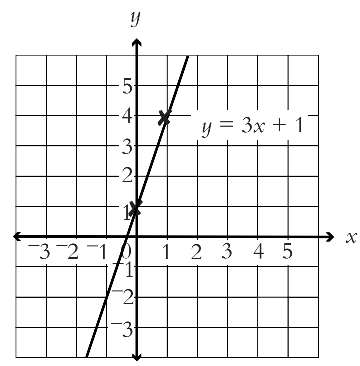
(b)



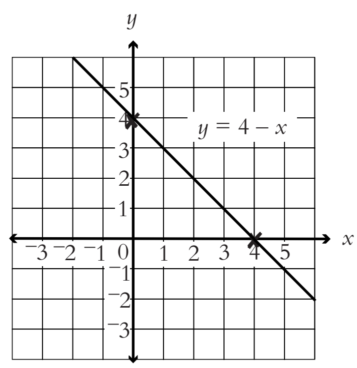
(c)



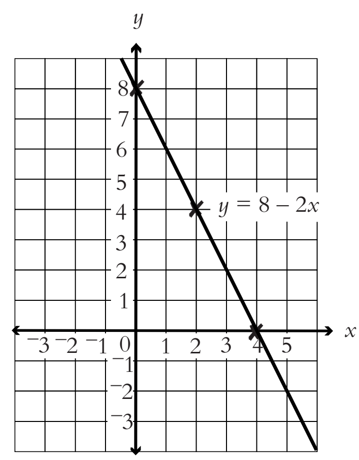
(d)



(d)



(f)



3. (a) 1 (b) 3 (c) 2 (d) 4

(e) $^{-}2$ (f) $\frac{1}{2}$ (g) $-\frac{1}{2}$

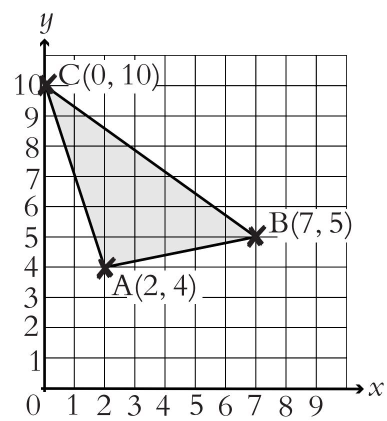
4. (a) $y = 4x + 2$ (b) $y = 2x - 5$

(c) $y = \frac{1}{2}x + 1$ (d) $y = {}^{-}x - 5$

5.

Equation	Gradient	Intercept
$y = 5x + 7$	5	7
$y = 3x - 2$	3	$^{-}2$
$y = {}^{-}3x + 2$	$^{-}3$	2
$y = {}^{-}4x - 2$	$^{-}4$	$^{-}2$
$y = {}^{-}2x + 3$	$^{-}2$	3
$y = \frac{1}{2}x + 1$	$\frac{1}{2}$	1
$y = {}^{-}x + 4$	$^{-}1$	4
$y = {}^{-}3x + 10$	$^{-}3$	10

6. (a)



(b) Gradient AB $= \frac{1}{5}$

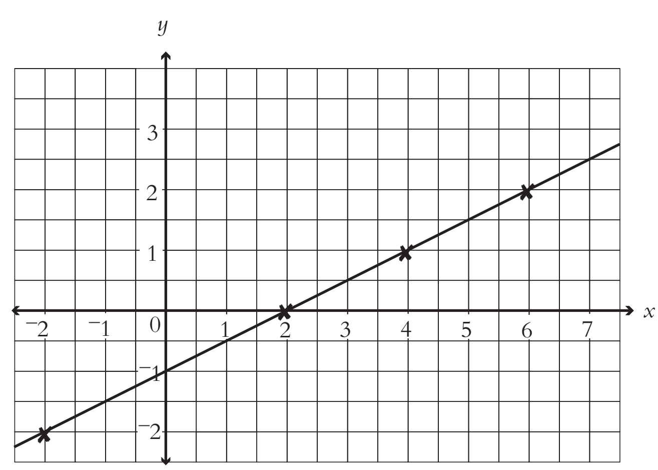
AC $= {}^{-}3$

BC $= -\frac{5}{7}$

7. (a) $y = 2x$ (b) $y = x - 4$ (c) $y = \frac{1}{2}x + 2$

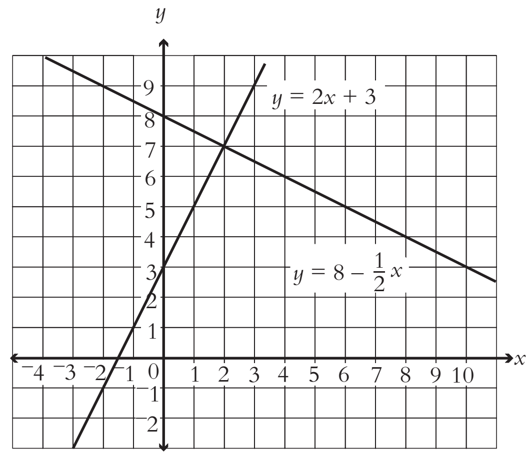
(d) $y = 2 - x$ (e) $y = 4x - 3$ (f) $y = \frac{-1}{3}x - 2$

8. (a)



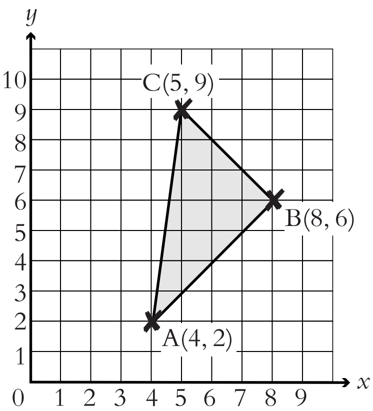
(b) $y = \frac{1}{2}x - 1$

9. (a)



(b) The lines cross at (2, 7)

10. (a)



(b) AB has equation $y = x - 2$

BC has equation $y = 14 - x$

AC has equation $y = 7x - 26$

(Pg. 50) Skill Exercises: Solving Linear Equations 1

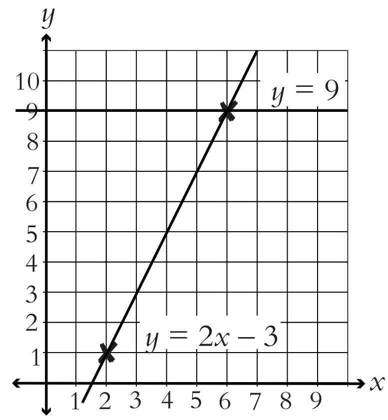
1. (a) $x = 8$ (b) $x = 11$ (c) $x = 3$ (d) $x = 30$
   (e) $x = 8$ (f) $x = 7$ (g) $x = 13$ (h) $x = 7$
   (i) $x = 40$ (j) $x = 500$ (k) $x = 32$ (l) $x = 25$

2. (a) $x = 4$ (b) $x = 7$ (c) $x = 3$ (d) $x = 2$
   (e) $x = 7$ (f) $x = 59$ (g) $x = 8$ (h) $x = 5$
   (i) $x = 11$ (j) $x = 10$ (k) $x = 3$ (l) $x = 2$

3. (a) $x = 2$ (b) $x = 5$ (c) $x = 4$ (d) $x = 3$
   (e) $x = 5$ (f) $x = 4$ (g) $x = 2\frac{1}{2}$ (h) $x = 4\frac{1}{2}$

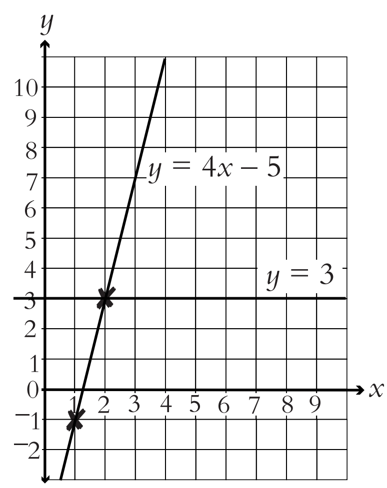
4. (a) $x = 3$ (b) $x = 6$ (c) $x = 1$

5.



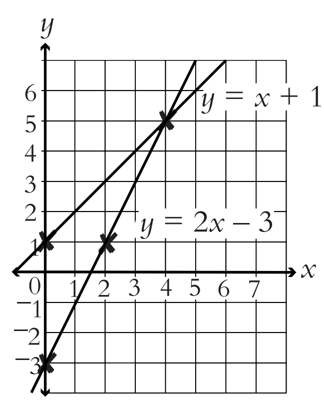
The solution is $x = 6$

6.



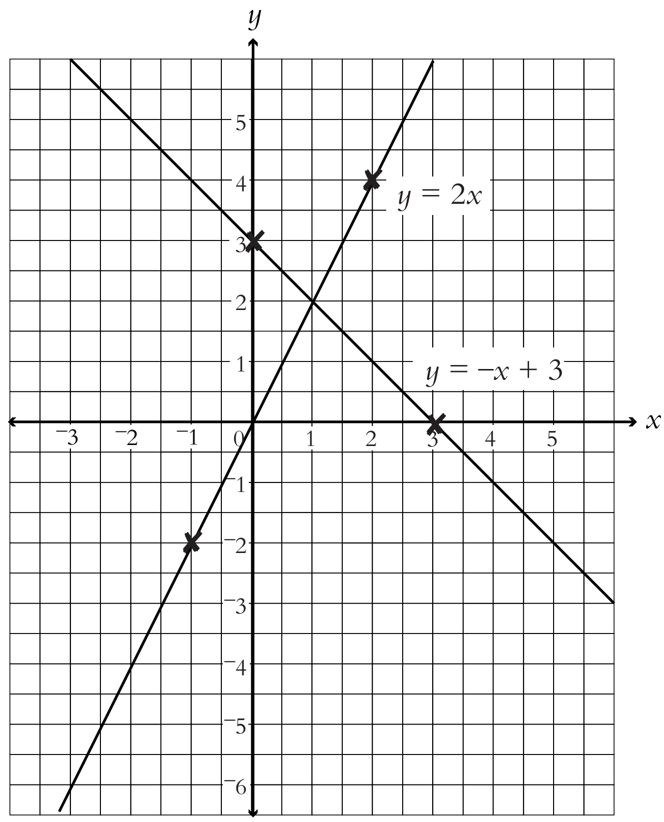
The solution is $x = 2$

7. (a)

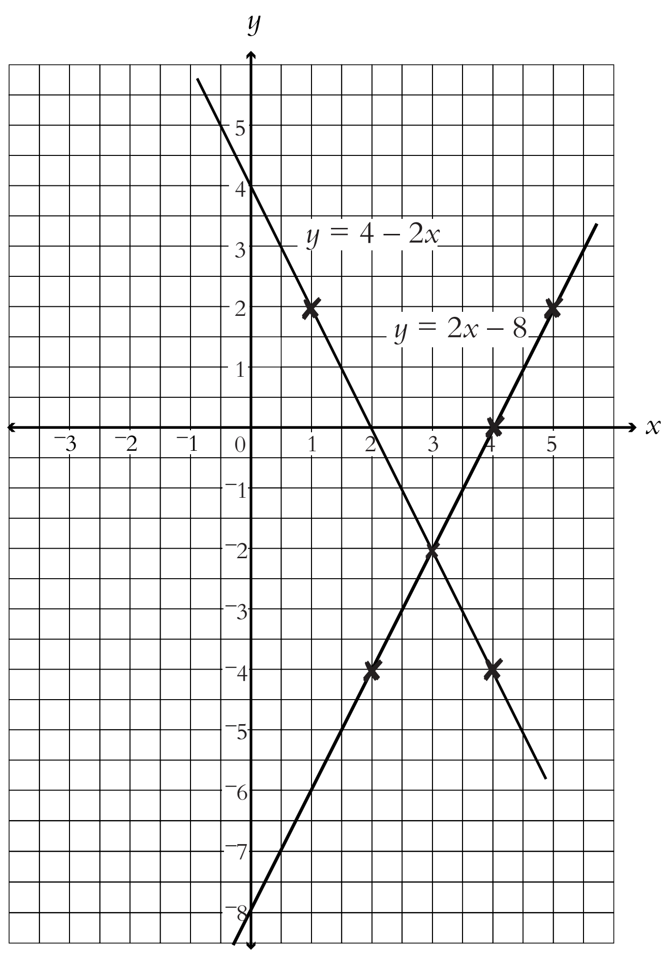


(b) The solution is $x = 4$

8. (a) $x = 1$ (see graph below).

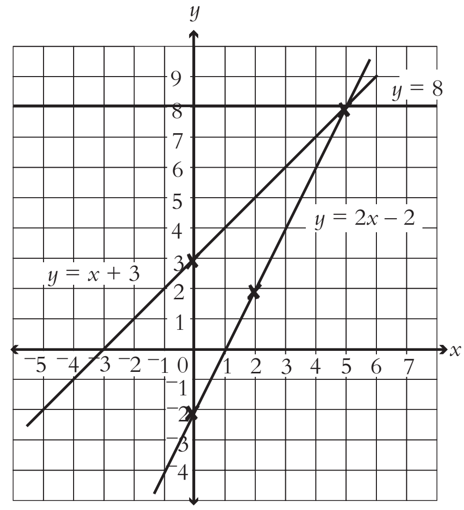


(b) $x = 3$ (see graph below).



9. (a) $x = 3$ (b) $x = 4$ (c) $x = 1$

10.



(a) $x = 5$

(b) $x = 5$

(c) $x = 5$

(Pg. 53) Skill Exercises: Listing Domains and Ranges

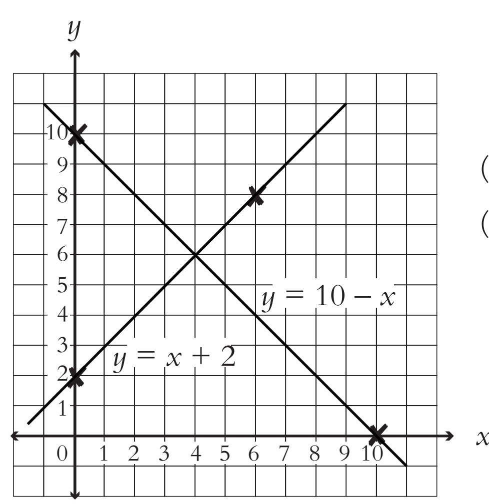
1. (a) Domain = $\{1, 3, 5\}$ Range = $\{2, 3, 4\}$
   (b) Domain = $\{1, 2, 3, 4\}$ Range = $\{5, 7\}$
   (c) Domain = $\{2, 3, 4, 5, 6\}$ Range = $\{3, 4, 5, 6, 7\}$
   (d) Domain = $\{^-1, ^-2, ^-3\}$ Range = $\{4\}$

2.

Relation	Domain	Range
		$\{1, 2, 3, 4\}$
	$\{3, 9, 11, 15\}$	
		$\{2, 3, 4, 5, 6\}$
$y = x + 5$		
		$\{^-5, ^-3, ^-1, 1, 3\}$

(Pg. 59) Skill Exercises: Solving Simultaneous Equations

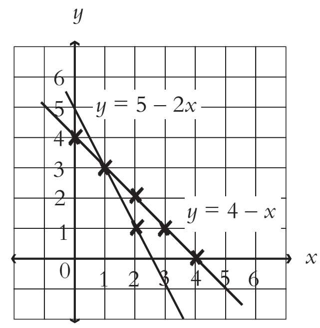
1. (a)



(b) Intersection (4, 6)

(c) $x = 4$, $y = 6$

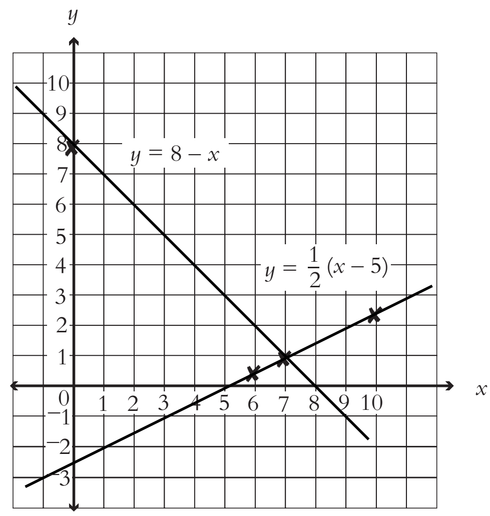
2. (a)



(b) Intersection (1, 3)

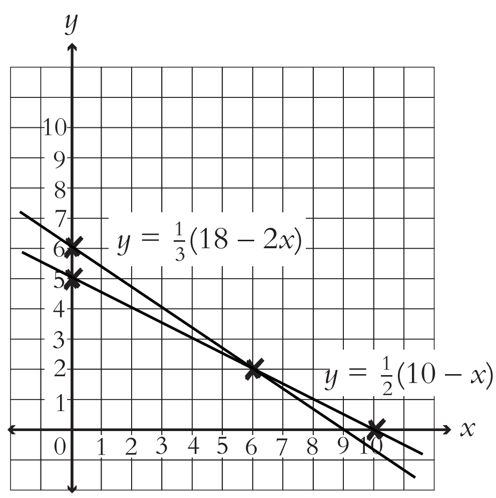
(c) $x = 1$, $y = 3$

3.



The solution is $x = 7$, $y = 1$

4.

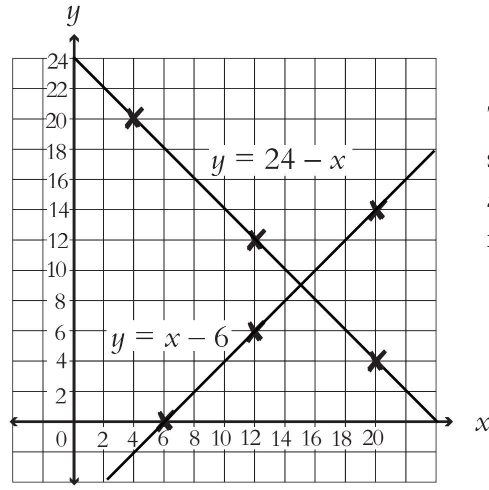


The solution is $x = 6$, $y = 2$

5. (a) $x + y = 24$

$x - y = 6$

(b)



The solutions to the simultaneous equations are $x = 15$, $y = 9$, so the numbers are 15 and 9.

6. $x + 2y = 8$ and $2x + y = 10$ or $y = {}^{-}2x + 10$

$y = \frac{-1}{2}x + 4$

7. (a) Because it eliminates the unknown x, leaving an equation with only one unknown (y).

(b) $3y = 6$

$y = 2, x = 3$

8. (a) $x = 3, y = 1$ (b) $x = 5, y = 2$ (c) $x = 7, y = 2$

(d) $x = 11, y = {}^{-}4$ (e) $x = 9, y = 3$ (f) $x = 10, y = 3$

9. (a) Because it eliminates x, leaving an equation with only one unknown (y)

(b) $x = 11, y = 1$

10. (a) $x = 1, y = 3$ (b) $x = 5, y = 3$ (c) $x = 5, y = 1$

(d) $x = 7, y = 1$ (e) $x = 20, y = 2$ (f) $x = 7, y = 2$

(Pg. 62) Skill Exercises: Simplifying Algebraic Expressions

1. (a) $6a$ (b) $3a + 12$ (c) $11a + 6b$
(d) $4x + 11y$ (e) $13x - y$ (f) $2a + 10b$
(g) $3a + b + 6$ (h) $6p - 7q$ (i) $^{-}7x + 3y$
(j) $2x - p$ (k) $15x - 9z$ (l) $^{-}14x + 5z$
(m) $8a + 4q - 6x$ (n) $^{-}p - q + x + z$ (o) $3x - 2y + 10$
(p) $21x - 32q$ (q) $2y$ (r) $2x$
(s) $^{-}19x + 11y$ (t) $23x - 18y - 4$ (u) $^{-}7x - 10y$
(v) $^{-}4p + 4q$

2. (a) $6x^2 + 8x$ (b) $x^2 + 13x + 10$ (c) $2x^2 + 10x$
(d) $5x^2 + 2x + 10$ (e) $^{-}x^2 + 7x$ (f) $3x^2 + 4y^2$
(g) $2x^2 + y^2 - x - y$ (h) $5x^2 - 3x - 10$ (i) $3x^2 - 3y^2 - x - y$
(j) $5x^2 + 2y - 4$ (k) $5ab + \mathrm{cd}$ (l) $2xy + 5xz$
(m) $11ab - 3a\mathrm{d}$ (n) $9pq - 3q\mathrm{r}$

3. (a) $3x + 15$ (b) $24 + 4x$ (c) $7x + 14$
(d) $2x + 12$ (e) $5x + 10$ (f) $8x + 12$
(g) $15x + 10$ (h) $40x + 24$ (i) $7x - 42$
(j) $40 - 8x$ (k) $8x - 28$ (l) $35x - 21$
(m) $18x - 30$ (n) $4x - 8y$ (o) $5x + 10y + 15z$
(p) $5x + x^2$ (q) $2a - a^2$ (r) $4b - 12$
(s) $2x^2 - 12x$ (t) $8x^2 + 12x$ (u) $21x - 6x^2$
(v) $8x^2 - 40x$

4. (a) $7a + 26$ (b) $7x + 23$ (c) $8x + 22$
(d) $2x^2 + 7x$ (e) $x^2 + 9x + 4$ (f) $12a + 17b$

(Pg. 66) Skill Exercises: Solving Linear Equations 2

1. (a) $x = 14$ (b) $x = 9$ (c) $x = 14$ (d) $x = 8$
   (e) $x = 4$ (f) $x = 1\frac{1}{3}$ (g) $x = {}^-4$ (h) $x = {}^-1$
   (i) $x = \frac{{}^-5}{3}$ (j) $x = 7$ (k) $x = 6$ (l) $x = {}^-2$
   (m) $x = 10$ (n) $x = 1$ (o) $x = \frac{3}{5}$ (p) $x = {}^-2\frac{1}{2}$
   (q) $x = \frac{1}{5}$ (r) $x = \frac{{}^-3}{4}$ (s) $x = {}^-3$ (t) $x = 0$
   (u) $x = \frac{{}^-2}{5}$ (v) $x = 36$ (w) $x = 15$ (x) $x = 5$

2. (a) $x = {}^-3$ (b) $x = {}^-6$ (c) $x = \frac{13}{4}$ (d) $x = 14$
   (e) $x = 4$ (f) $x = 10$ (g) $x = 1$ (h) $x = {}^-3$
   (i) $x = 13$ (j) $x = 3$ (k) $x = 4$ (l) $x = \frac{2}{3}$
   (m) $x = 3$ (n) $x = 2$ (o) $x = \frac{5}{3}$ (p) $x = 4$
   (q) $x = 2$ (r) $x = \frac{1}{2}$ (s) $x = 8$ (t) $x = 12$
   (u) $x = 32$ (v) $x = 14$ (w) $x = 39$ (x) $x = 12$

3. (a) $(3x + 40) + 80 + x = 360$; $x = 60°$
   (b) $(x + 10) + (x + 30) + (2x + 30) + x = 360$; $x = 58°$
   (c) $(x + 10) + (x + 30) + (x - 10) = 360$; $x = 110°$
   (d) $(140 - x) + (120 - x) + (160 - x) = 360$; $x = 20°$

4. $x + x + (x + 1) + (x + 1) = 10$; $x = 2$ m

5. $2x + 10 = 42$; $x = 16$

6. $6x - 8 = 34$; $x = 7$

7. $4m + 2 = 6$; $m = 1$;
   Mele drives for one hour; Sione drives for three hours and Filipo drives for two hours.

8. (a) $80 + (60 \times x) = 290$; $x = 3\frac{1}{2}$ (3 hours and 30 minutes)
   (b) $80 + (60 \times x) = 220$; $x = 2\frac{1}{3}$ (2 hours and 20 minutes)

9. (a) $(x + 6) \times 2 = 18; \quad x = 3$
   (b) $(x \div 2) + 10 = 16; \quad x = 12$
   (c) $[(x \div 2) + 2] \times 2 = 9; \quad x = 5$
   (d) $[(x - 7) \div 2] \times 10 = 115; \quad x = 30$

10. $x + (x + 1) + (x + 2) + (x + 3) = 114; \quad x = 27$

11. 6

12. (a) $y = 2x + 1$ (b) $x = 4$

13. (a) 7 (b) 4

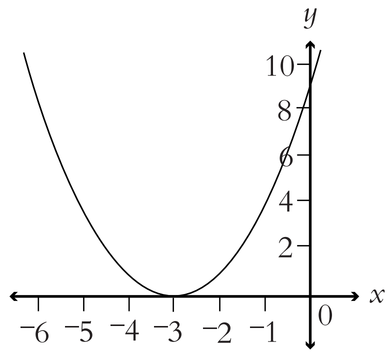
14. $x = 2$

15. (a) $x = 4\frac{1}{2}$ (b) $y = 4$

(Pg. 71) Skill Exercises: Solving Quadratic Equations

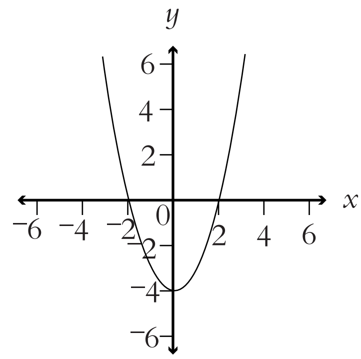
1. (a) $x = {}^-4$ or 3 (b) $x = 5$ or ${}^-3$ (c) $x = {}^-6$ or 2
   (d) $x = 6$ or 0 (e) $x = 0$ or $\frac{4}{3}$ (f) $x = 0$ or $\frac{9}{4}$
   (g) $x = 3$ or ${}^-3$ (h) $x = 7$ or ${}^-7$ (i) $x = \frac{8}{3}$ or $\frac{-8}{3}$
   (j) $x = 4$ (both answers) (k) $x = {}^-5$ (both answers)
   (l) $x = 6$ or ${}^-3$ (m) $x = 4$ or 7 (n) $x = 6$ or ${}^-5$
   (o) $x = 10$ or 4 (p) $x = {}^-3$ or $\frac{-1}{2}$ (q) $x = \frac{3}{2}$ or ${}^-4$
   (r) $x = 1$ or $\frac{4}{3}$ (s) $x = \frac{3}{4}$ or ${}^-1$ (t) $x = \frac{1}{2}$ or ${}^-3$
   (u) $x = 7$ or $\frac{5}{2}$

2. (a)



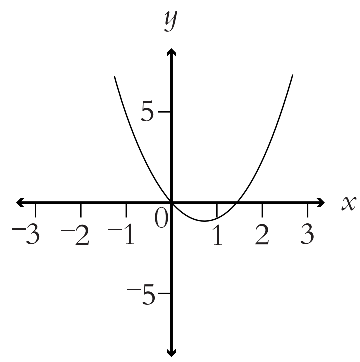
The curve touches the x-axis at the point where $x = -3$

(b)



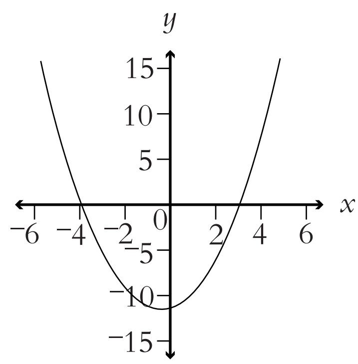
The curve cuts the x-axis at the points where $x = 2$ and $x = -2$

(c)



The curve cuts the x-axis at the points where $x = 0$ and $x = \frac{3}{2}$

(d)



The curve cuts the x-axis at the points where $x = 3$ and $x = -4$

3. (a) $(x + 4)(x - 4) = 0, \quad \therefore x = -4$ or 4
   (b) $(x + 25)(x - 25) = 0, \quad \therefore x = -25$ or 25
   (c) $(x - 1)(x + 1) = 0, \quad \therefore x = -1$ or 1

4. (a) $x = 5$ (b) $x = 4$ (c) $x = 4$ (d) $x = 8$

5. at $h = 0, \quad t = 0.8$

6. (a) $L = \dfrac{2V}{W(E + R)}$ (b) $L = 5$ or $L = 10$

(Pg. 76) Skill Exercises: Plotting Quadratic Functions

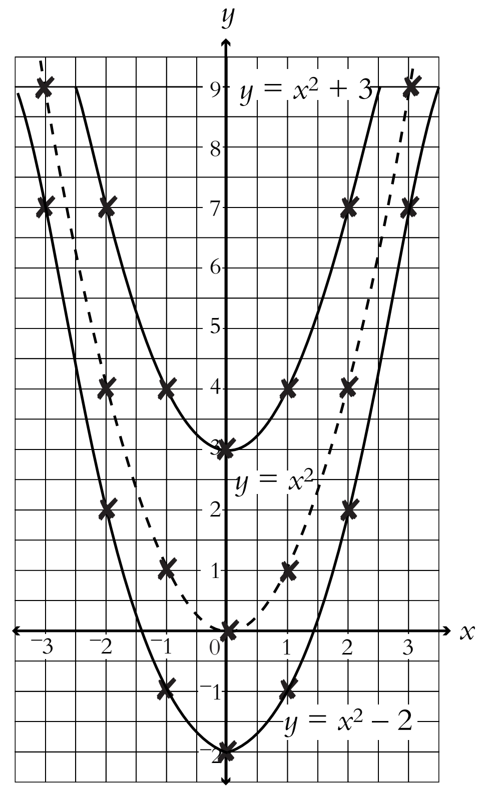
1. (a), (b), (c) graphs.

   (d) $y = x^2 \rightarrow y = x^2 + 3$

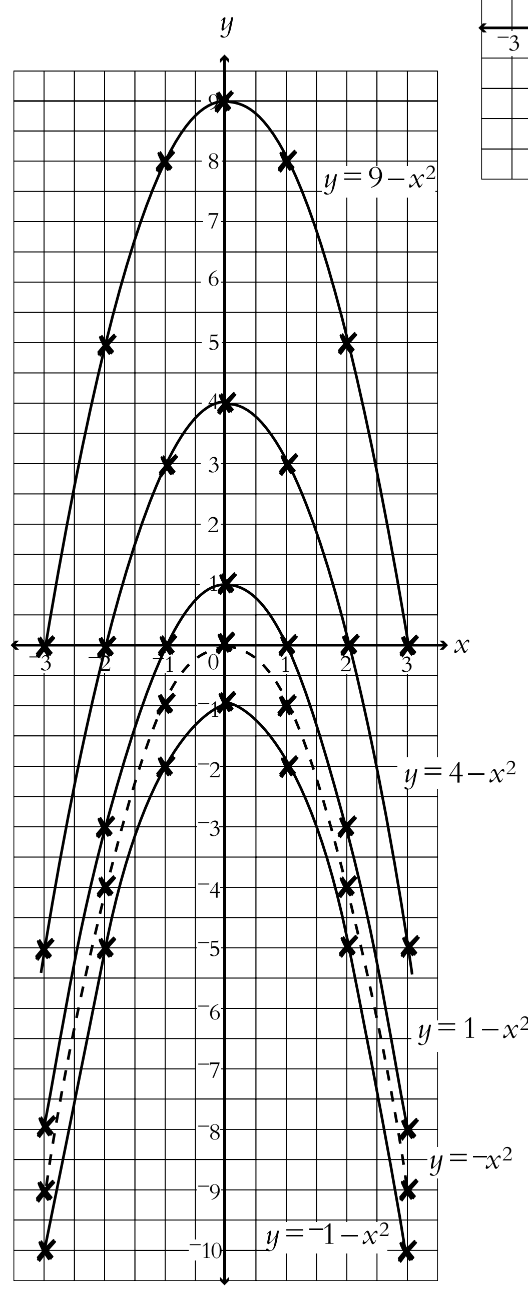
   translation $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$

   $y = x^2 \rightarrow y = x^2 - 2$

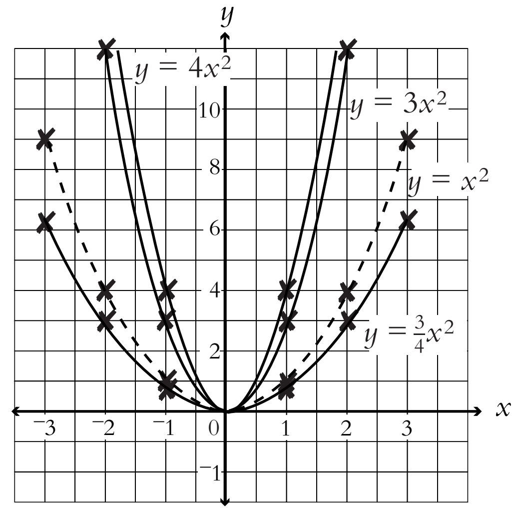
   translation $\begin{bmatrix} 0 \\ -2 \end{bmatrix}$



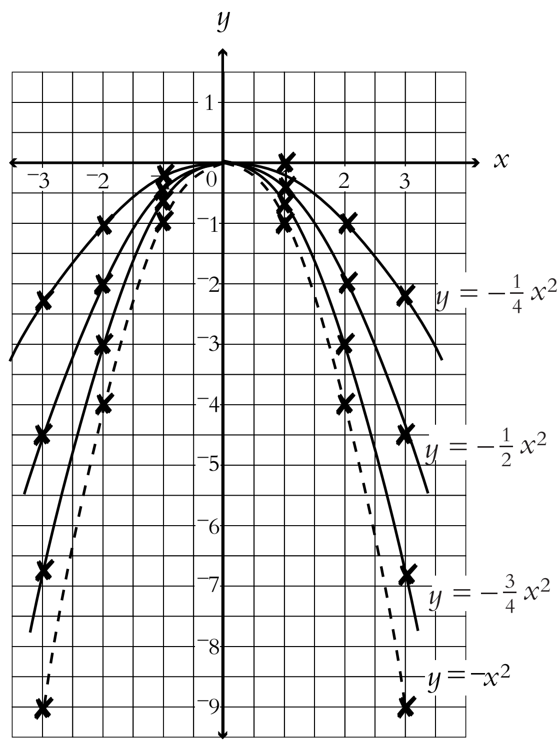
2.



3.

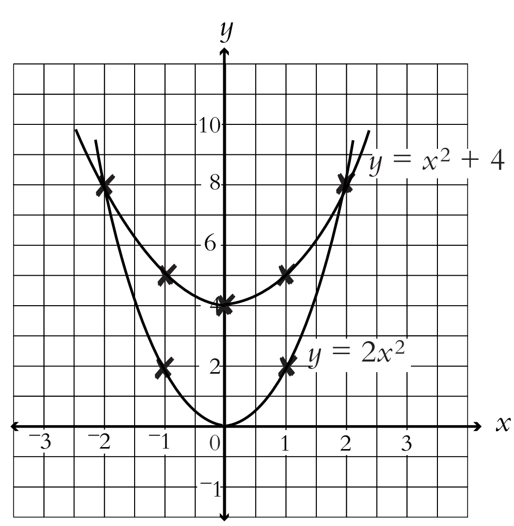

y
x
$y = 4x^2$
$y = 3x^2$
$y = x^2$
$y = \frac{3}{4}x^2$

4.

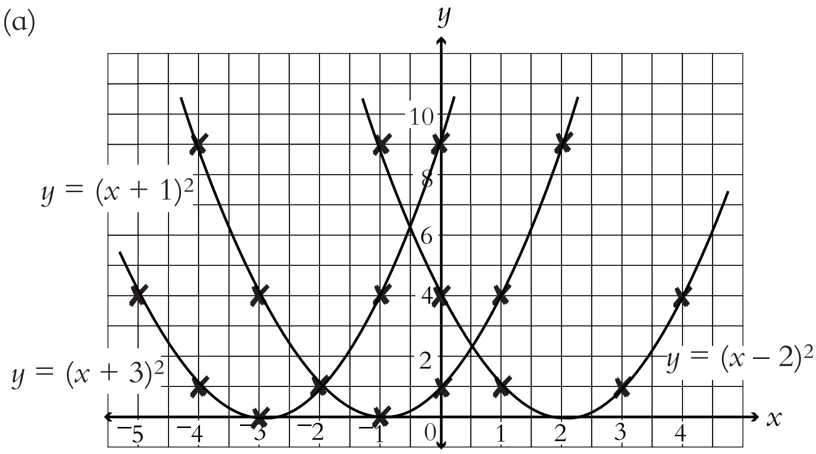

y
x
$y = -\frac{1}{4}x^2$
$y = -\frac{1}{2}x^2$
$y = -\frac{3}{4}x^2$
$y = ^-x^2$

5. (a)

(b) $(^-2, 8)$, $(2, 8)$


y
x
$y = x^2 + 4$
$y = 2x^2$

6. (a)

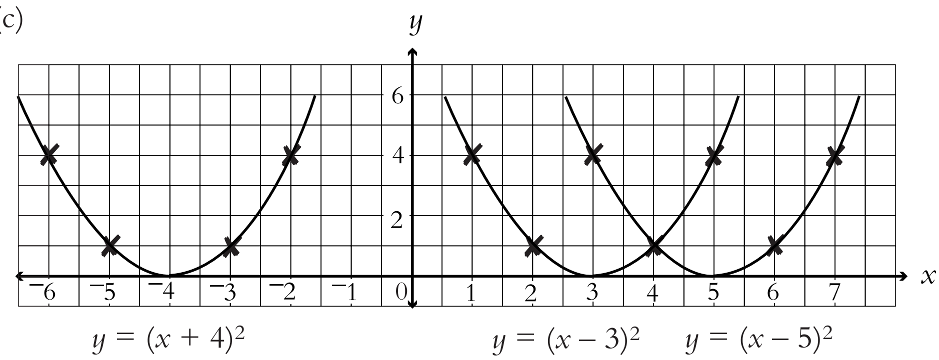


(b) $y = x^2 \rightarrow y = (x + 1)^2$ translation $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

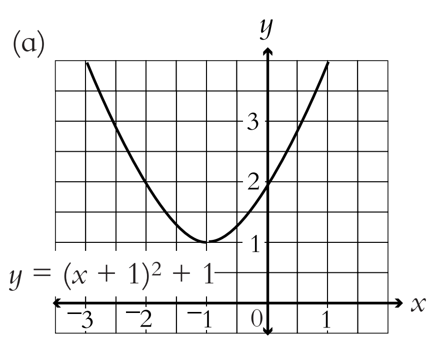
$y = x^2 \rightarrow y = (x + 3)^2$ translation $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$

$y = x^2 \rightarrow y = (x - 2)^2$ translation $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$

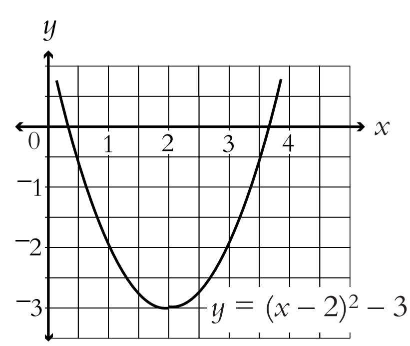
(c)



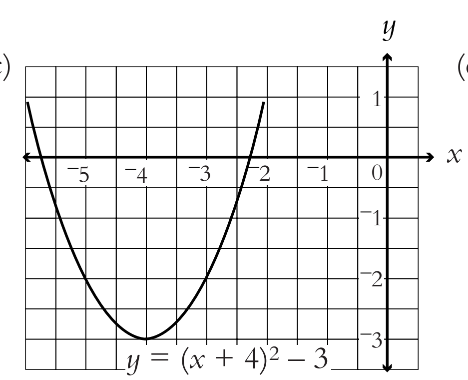
7. (a)



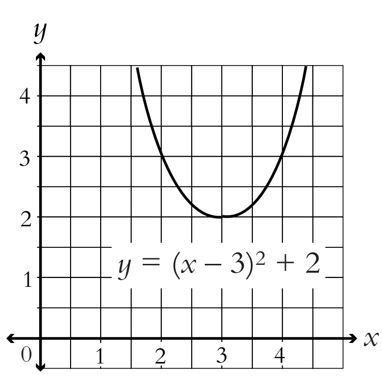
(b)



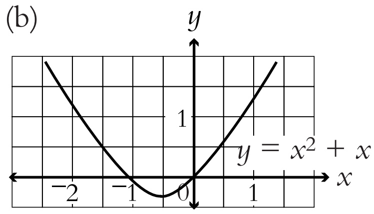
(c)



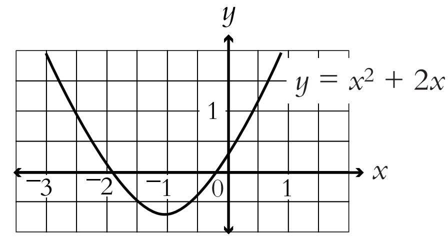
(d)



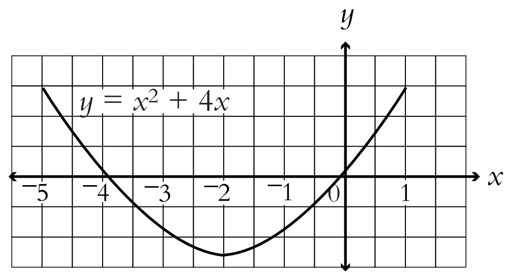
8. (a), (b)



Lowest point $\left(-\frac{1}{2}, -\frac{1}{4}\right)$

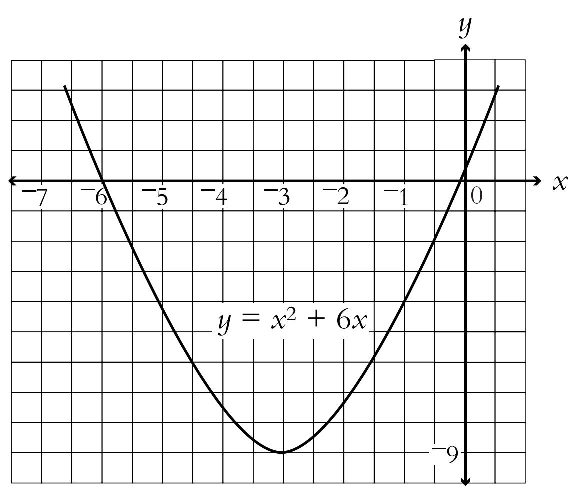


Lowest point $(^-1, ^-1)$



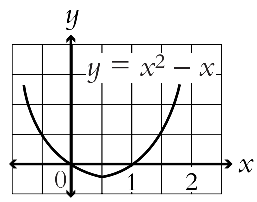
Lowest point $(^-2, ^-4)$

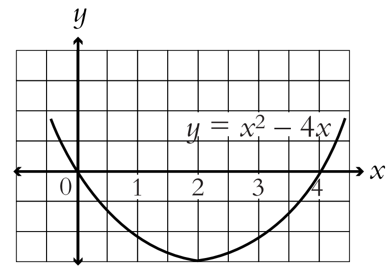
Lowest point of $y = x^2 + bx$ is $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$

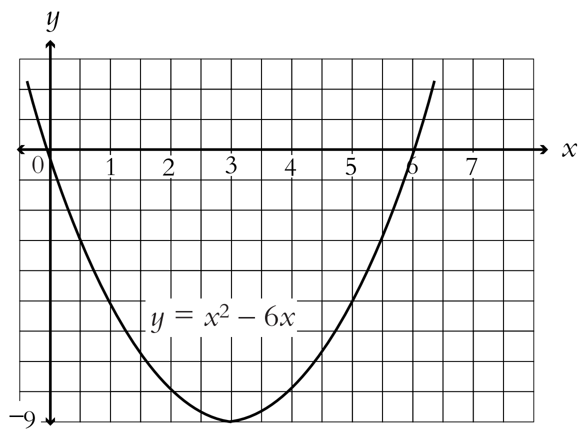


Lowest point $(^-3, ^-9)$

(c)

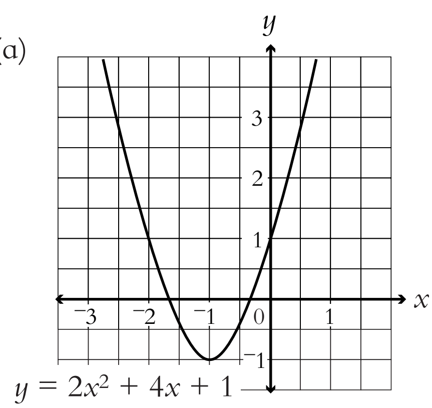


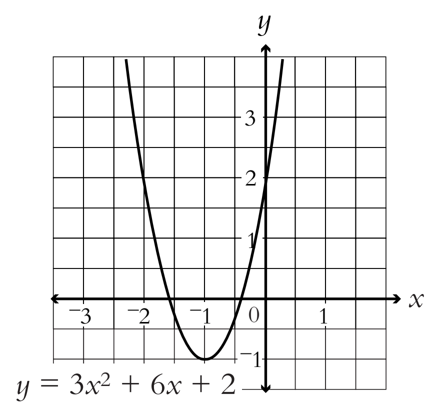




(d) $\left(\frac{b}{2}, -\frac{b^2}{4}\right)$

9. (a)





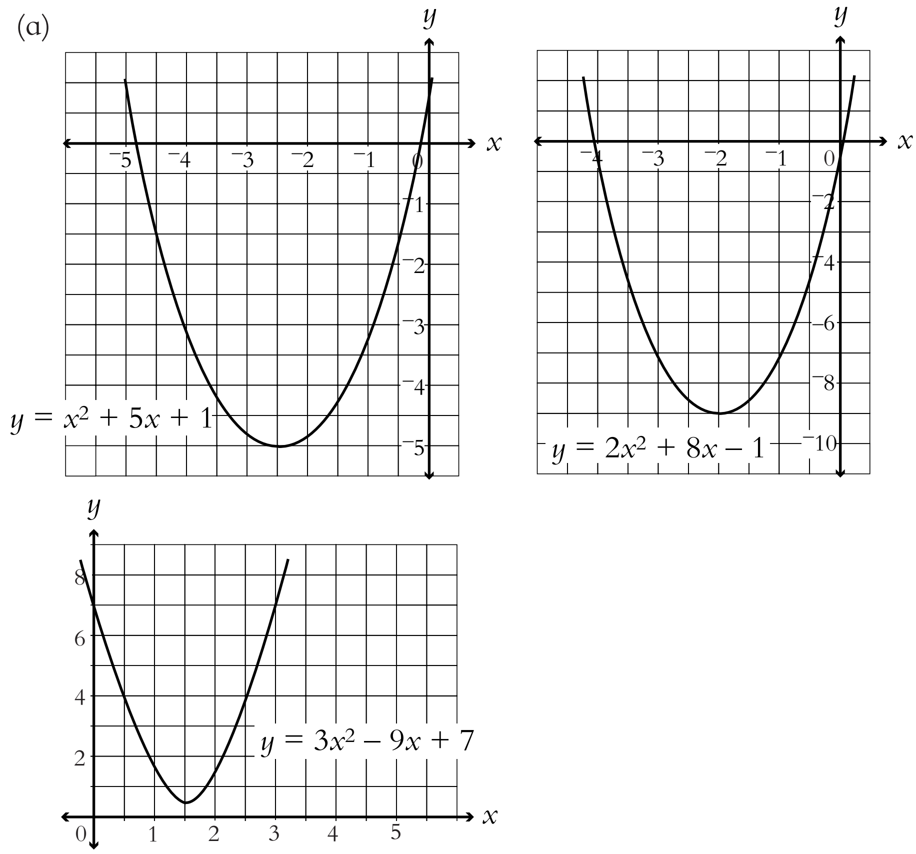
(b) (0, 1) (0, 2)

(c) $(0, c)$

(d) $(^-1, ^-1)$ $(^-1, ^-1)$

(e) $(^-1, c - a)$

10. (a)



(b) $\left(-\dfrac{b}{2a}, \dfrac{4ac - b^2}{4a}\right)$

11. (a) c is the y-intercept (or value of y when $x = 0$) and the graph shows $y = 10$ when $x = 0$

(b) $a = \dfrac{-1}{30}$; $b = 2$

Unit 4: ANSWERS — MEASUREMENT

Section 4.1 Working With Surface Areas

(Pg. 83) Skill Exercises: Calculating Surface Areas of Solid Figures

1. (a) 96 cm^2 (b) 136 cm^2 (c) 236 cm^2
   (d) 62 cm^2 (e) 250 cm^2 (f) 30.4 cm^2

2. (a) 108.38 cm^2 (to 2 d.p.) (b) 283 cm^2 (to the nearest cm^2)
   (c) 207 cm^2 (to the nearest cm^2) (d) 69 m^2 (to the nearest m^2)
   (e) 12.6 m^2 (to 1 d.p.) (f) 115 cm^2 (to the nearest cm^2)

3. The surface area of each cylinder is 301.44 cm^2 when $\pi = 3.14$, and 301.60 using the π key on your calculator).
   (a) V = 402 cm^3 (b) V = 276 cm^3 (c) V = 368 cm^3

4. The volume of each cuboid is 64 cm^3. The surface area of each cuboid is:
   (a) 96 cm^2, which is the smallest area
   (b) 112 cm^2
   (c) 168 cm^2

5. (a) 13 195 cm^2 (to the nearest cm^2) (2π rh)
   (b) 35 000 cm^2 (5m = 100cm $\therefore$ 500 × 70 = 35 000 m^2)

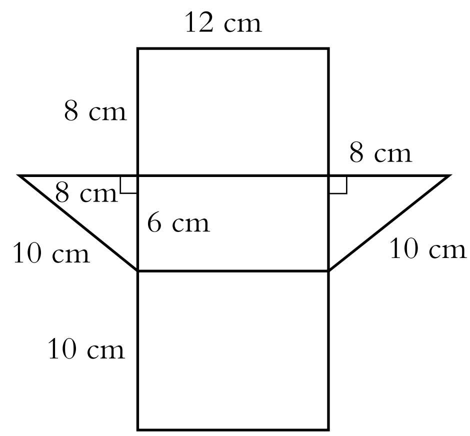
6. $6 \times 7^2 = 294$ cm^2 (cube length is 7cm)

7. $\left(\sqrt{\frac{150}{6}}\right)^3 = 5^3 = 125$ cm^3

8. Remember the tray does not have a top and the sleeve has no ends.

   (a) 54 cm^2 (b) 66 cm^2 (c) 120 cm^2

9.



A = 336 cm^2

10. 11 027 cm^2

    [Total surface area = $(2 \times 25\pi \times 12) + 2(40^2\pi - 25^2\pi) + (2 \times 40\pi \times 12)$]