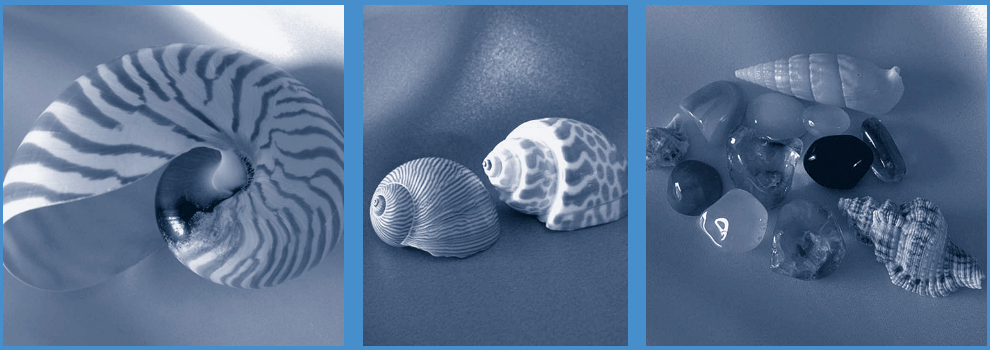
Book 3

Year 10



Mathematics

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Year 10 Book Three

GOVERNMENT OF SĀMOA
MINISTRY OF EDUCATION, SPORTS AND CULTURE

Designed, edited and typeset by Egan-Reid Ltd, Auckland as part of the Sāmoa Secondary Education Curriculum and Resources Project for:

Reprinted 2004 with minor amendments.

Funded by the New Zealand Agency for International Development,
Nga Hoe Tuputupu-mai-tawhiti.

Printed through Egan-Reid Ltd.

Managing Contractor: Auckland UniServices Limited.

ISBN 982–517–035–2

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In this unit you will be:

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- Average Speed.
- Other Rates.

8.2 Calculating and Comparing Unit Costs

Section 8.1 Calculating And Using Rates

Average Speed

When a bus is travelling between two villages, its speed will change. Sometimes it goes fast, then slow or it may stop. The average speed can be calculated if we know the total distance travelled and the total time taken.

$$\text{Average speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

If a bus travels 50 kilometers in two hours

$$\text{Average speed} = \frac{50 \text{ km}}{2 \text{ h}}$$

$$= 25 \text{ km/h}$$

The bus does not travel at a constant speed of 25 km/h. Its speed varies during the journey between 0 km/h and, perhaps, 30 km/h. The speed at any one time is called the instantaneous speed.

The following table lists units in common use for speed and their abbreviations:

DISTANCE	TIME	SPEED	ABBREVIATION
Mile	hours	miles per hour	mph
Kilometres	hours	kilometres per hour	km/h
Metres	hours	metres per hour	m/h
Metres	seconds	metres per second	m/s
Centimetres	seconds	centimetres per second	cm/sec or cm/s

Example 1

Mele drives from Salelologa to Asau, a distance of 80 kilometres, in two and a half hours. She then drives from Asau to Palauli Secondary School, a distance of 65 kilometres, in one hour and 30 minutes.

Determine her average speed for each journey.

Solution

Salelologa to Asau

$$\text{Average speed} = \frac{80 \text{ kilometres}}{2.5 \text{ hours}}$$

$$= 32 \text{ kilometres/hour}$$

Asau to Palauli

$$\text{Average speed} = \frac{65 \text{ kilometres}}{1.5 \text{ hours}}$$

$$= 43.33 \text{ kilometres/hour}$$

Example 2

Sione can type 960 words in 20 minutes.

Calculate his typing speed in:

(a) words per minute,

(b) words per hour.

Solution

(a) $$\text{Typing speed} = \frac{960}{20}$$

$$= 48 \text{ words per minute}$$

(b) $$\text{Typing speed} = 48 \times 60$$

$$= 2880 \text{ words per hour}$$

Skill Exercises: Average Speed

1. Pita drives 320 kilometres in eight hours. Calculate his average speed.
2. Susi drives from Faleolo to Aleipata, a distance of 90 kilometres, in four hours. Calculate her average speed.
3. A snail moves 8 m in 2 hours. Calculate the average speed of the snail in metres per hour.

4. A New Zealand truck driver keeps a record of each journey he makes. Calculate the average speed for each journey, using the table below:

Start	Finish	Start Time	Finish Time	Distance
Auckland	Rotorua	0800	1200	172 kilometres
Rotorua	Wellington	1400	1900	280 kilometres
Wellington	Timaru	1000	1800	300 kilometres
Timaru	Dunedin	0700	0930	120 kilometres
Dunedin	Bluff	1030	1530	175 kilometres

5. Siaki takes $1\frac{1}{2}$ hours to drive 30 km. Calculate his average speed in km/h.

6. Rebecca cycles 20 kilometres on her bike in two hours and 30 minutes. Calculate her average speed in kilometres per hour.

7. Julie can type 50 words in two minutes. Debbie can type 300 words in 15 minutes. Calculate the typing speed of each girl in:

   (a) words per minute.

   (b) words per hour.

8. Salote, Ema and Ene each drive from Auckland to Hamilton, a distance of 100 kilometres. Salote takes 1 hour, Ema takes 2 hours and Ene takes $1\frac{1}{2}$ hours. Calculate the average speed for each of the drivers.

9. Eva is on holiday in Britain. She drives from Edinburgh to Dover in three stages:

	Start Time	Finish Time	Distance
Edinburgh to Leeds	0630	1000	350 kilometres
Leeds to London	1030	1300	330 kilometres
London to Dover	1500	1700	130 kilometres

   Calculate her average speed for each stage of her journey.

10. Silia drives 220 km in $3\frac{1}{2}$ hours. Calculate her average speed correct to the nearest km/h.

Other Rates

A rate is one quantity divided by another quantity with different units. In the last section we divided distance (in km) by time (in hours) to get the average speed.

There are other rates commonly used.

1. Rate of flow.

   This tells us how much liquid flows in a given time.

2. Hourly rate.

   This tells us how much money a worker earns every hour.

3. Exchange rate.

   This tells us how much foreign money we might get for one tala.

When you visit another country you need to change your money to the currency of the country you visit.

To change Samoan money to another currency, you must multiply by the exchange rate. This rate is given by the bank.

To change another currency into Samoan tala you must divide by the exchange rate.

Example 1

A water tank can hold 1000 ℓ (ℓ is the symbol for litres) of water when full. It is half full. Sina leaves the tap open at the bottom and the tank drains out in 10 hours. What is the rate of flow?

Solution

$$\begin{aligned}\text{Rate of flow} &= \frac{\text{volume}}{\text{time}}\\ &= \frac{500\ \ell}{10\ \text{h}}\\ &= 50\ \ell/\text{h}\end{aligned}$$

Example 2

Iosefa works eight hours a day for five days and earns $320. What is his hourly rate of pay?

Solution

$$\text{Hourly rate} = \frac{\text{total pay}}{\text{time in hours}}$$

$$= \frac{\$320}{40\text{ h}}$$

$$= \$8/\text{h}$$

Example 3

Misa is going to New Zealand. She has $150. She goes to the bank and finds that the exchange rate is 0.6970. How much New Zealand money will she get for her $150?

Solution

NZ Dollars = Samoan Tala × Exchange Rate

= $150 × 0.6970

= NZ$104.55

Misa will get about $104 New Zealand.

Skill Exercises: Other Rates

1. A hotel is filling its swimming pool, which can hold 600 000 ℓ of water. The pumps can deliver 50 000 ℓ of water per hour. How long will it take to fill the pool?

2. A taxi service has a fixed charge of $1.20, and then 78 c per km. Calculate the cost for journeys of the following lengths:
   (a) 1 km
   (b) 2 km
   (c) 4.5 km
   (d) 10.5 km

3. Aleki buys a 20 m length of fabric for $18.60.
   (a) What is the cost per metre of the fabric?
   (b) What would be the cost of 9.2 m of the fabric?

4. Five people work in a shop. The following table lists the hours worked and the total paid in one week:

	Hours Worked	Total Paid
Dee	8	\$28.64
Nadina	12	\$43.44
Lisa	42	\$302.40
Mary	38	\$136.80
Clare	35	\$134.40

   (a) Who is paid the most per hour?

   (b) Who is paid the least per hour?

5. A five litre tin of paint is used to paint a wall that measures 6.25 m by 4 m. Calculate the rate at which paint is applied to the wall in litres per m^2.

6. The Central Bank of Samoa advertises the following Exchange Rates:

Your Samoa Tala is worth overseas:

Australia	0.5499	Dollars
Europe	0.3201	Euro
Fiji	0.6452	Dollars
United States	0.2830	Dollars

   (a) Tasi is going to Los Angeles and has \$1250 (Samoan). How much is this worth in US dollars?

   (b) Tina is coming back from Sydney and has Australian \$1200. She takes this money to the bank in Apia. How many tala will she get in exchange?

Section 8.2 Calculating And Comparing Unit Costs

The unit cost tells us the price of one thing. If we know the unit costs when we are shopping then we can make sure we get the best deal.

Example

A 2.5 ℓ bottle of cola costs \$1.75 in the supermarket but a 1 ℓ bottle of cola costs \$0.80. Which is the better buy?

Solution

The 2.5 ℓ bottle

$$\text{Cost per litre} = \frac{\$1.75}{2.5\ \ell}$$

$$= \$0.70 \text{ per litre}$$

The 1.0 ℓ bottle

$$\text{Cost per litre} = \frac{\$0.80}{1\ \ell}$$

$$= \$0.80 \text{ per litre}$$

The 2.5 ℓ bottle of cola is the better buy because it is \$0.10 cheaper per litre.

Skill Exercises: Calculating and Comparing Unit Costs

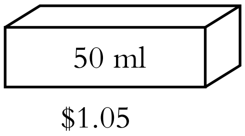
1. Two bottles of washing detergent are shown:

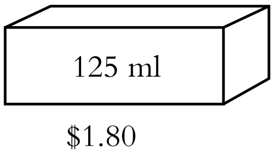
   (a) What is the cost per litre of the two brands?

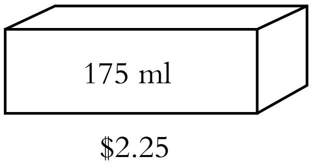
   (b) Which one is the better value for money if they are both the same to use?

2. In Shop A 1.5 kg of rice costs 57 sene. In Shop B, 2 kg of the same rice costs 79 sene. Which shop gives better value for money?

3. Keleti wants to buy some toothpaste. Which size, 50 ml, 125 ml or 175 ml is the best value for money?







Unit 9: ALGEBRA – PART 2

In this unit you will be:

9.1 Drawing Linear Graphs

- Plotting Co-ordinates.
- Plotting Points on a Straight Line.
- Drawing Graphs given their Equations.
- Finding the Equation of a Straight Line.

9.2 Expanding and Factorising Expressions

- Expanding Expressions with Two Brackets.
- Factorising Expressions.

Section 9.1 Drawing Linear Graphs

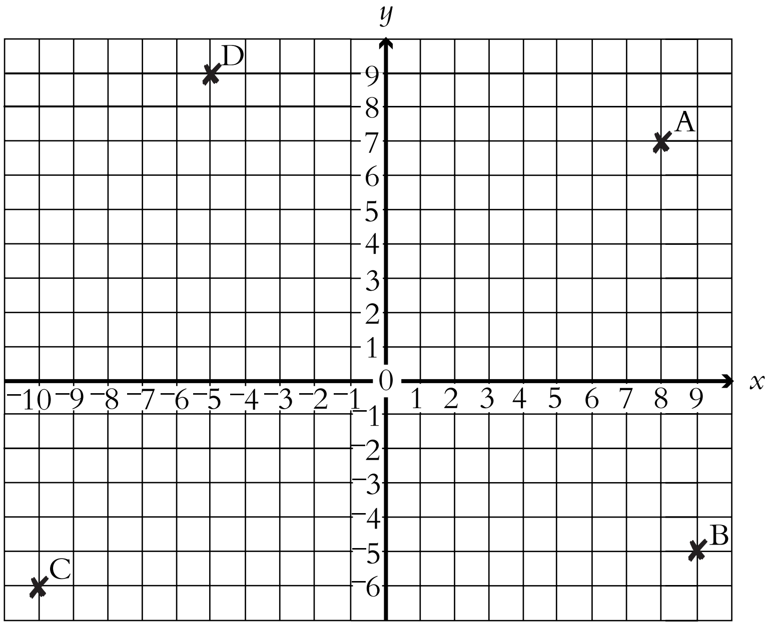
A linear graph is a straight line graph. The points on the line are called co-ordinates.

Plotting Co-ordinates

In this section, co-ordinates are given in pairs. The first number in the pair is the x co-ordinate and the second number is the y co-ordinate.

Example 1

What are the co-ordinates of the points marked on the following grid:



Solution

The co-ordinates are:

A	(8, 7)
B	(9, ⁻5)
C	(⁻10, ⁻6)
D	(⁻5, 9)

Example 2

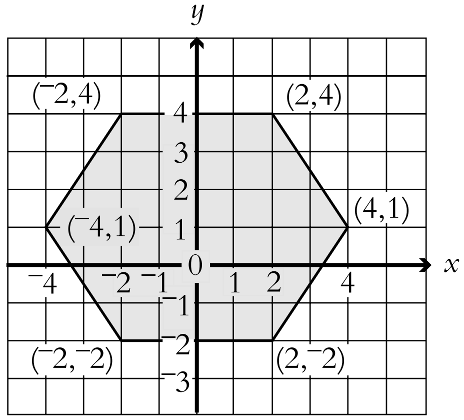
The co-ordinates of the corners of the shape are (2, 4), (4, 1), (2, ⁻2), (⁻2, ⁻2), (⁻4, 1) and (⁻2, 4).

(a) Draw the shape.

(b) What is the name of the shape?

Solution

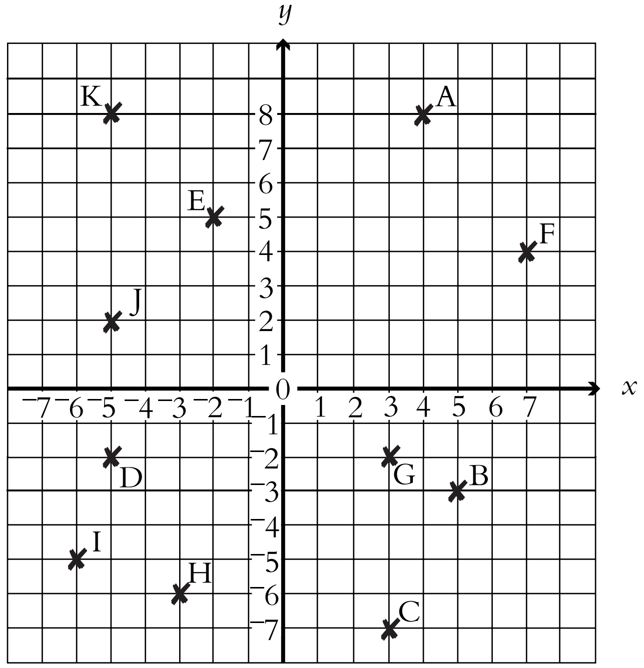
(a)



(b) The shape has six sides and is called a hexagon.

Skill Exercises: Plotting Co-ordinates

1. Write down the co-ordinates of each of the points marked on the following axes:



2. (a) Plot the points with co-ordinates (3, ⁻2), (⁻1, 6) and (⁻5, ⁻2).

   (b) Join the points to form a triangle.

   (c) What type of triangle have you drawn?

3. (a) Plot the points with co-ordinates $(^{-}1, 4)$, $(2, 5)$, $(5, 4)$ and $(2, ^{-}1)$.

   (b) Join these points, in order to form a shape.

   (c) What is the name of the shape?

4. The co-ordinates of three corners of a square are $(3, 1)$, $(^{-}1, 1)$ and $(3, ^{-}3)$. What are the co-ordinates of the other corner?

5. The co-ordinates of three corners of a rectangle are $(^{-}1, 6)$, $(^{-}4, 6)$ and $(^{-}4, ^{-}5)$. What are the co-ordinates of the other corner?

6. A shape has corners at the points with co-ordinates $(3, ^{-}2)$, $(6, 2)$, $(^{-}2, 2)$ and $(^{-}5, ^{-}2)$.

   (a) Draw the shape.

   (b) What is the name of the shape?

7. A shape has corners at the points with co-ordinates $(3, 1)$, $(1, ^{-}3)$, $(3, ^{-}7)$ and $(5, ^{-}3)$.

   (a) Draw the shape.

   (b) What is the name of the shape?

8. (a) Join the points with the co-ordinates below, in order to form a polygon:

   $(^{-}5, 0)$, $(^{-}3, 2)$, $(^{-}1, 2)$, $(1, 0)$, $(1, ^{-}2)$, $(^{-}1, ^{-}4)$, $(^{-}3, ^{-}4)$ and $(^{-}5, ^{-}2)$.

   (b) What is the name of the polygon?

9. Three of the corners of a parallelogram have the co-ordinates $(1, 5)$, $(4, 4)$ and $(6, ^{-}3)$.

   (a) Draw the parallelogram.

   (b) What are the co-ordinates of the other corner?

10. Ben draws a pattern by joining, in order, the points with the following co-ordinates:

    $(^{-}2, 1)$, $(^{-}2, 2)$, $(0, 2)$, $(0, ^{-}1)$, $(^{-}4, ^{-}1)$, $(^{-}4, 4)$, $(2, 4)$ and $(2, ^{-}3)$.

    What are the co-ordinates of the next three points he would use?

Plotting Points on a Straight Line

In this section we plot points that lie on a straight line, and look for relationships between the co-ordinates of these points.

Example 1

(a) Plot the points with co-ordinates:

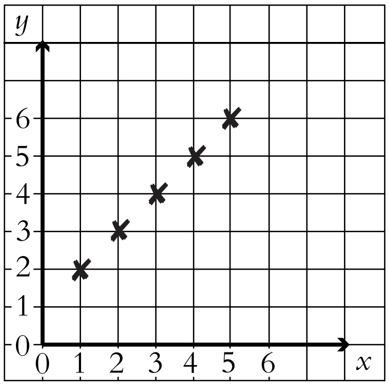
(1, 2), (2, 3) (3, 4) (4, 5) and (5, 6).

(b) Draw a straight line through these points.

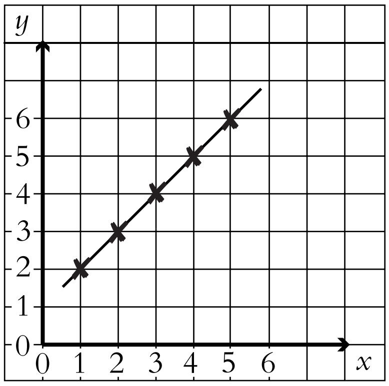
(c) Describe how the x and y co-ordinates of these points are related.

Solution

(a) The points are plotted below:



(b) A straight line can be drawn through these points.



(c) The y co-ordinate is always one more than the x co-ordinate, therefore we can write $y = x + 1$.

Example 2

(a) Plot the points with co-ordinates: (0, 0), (1, 3), (3, 9) and (5, 15).

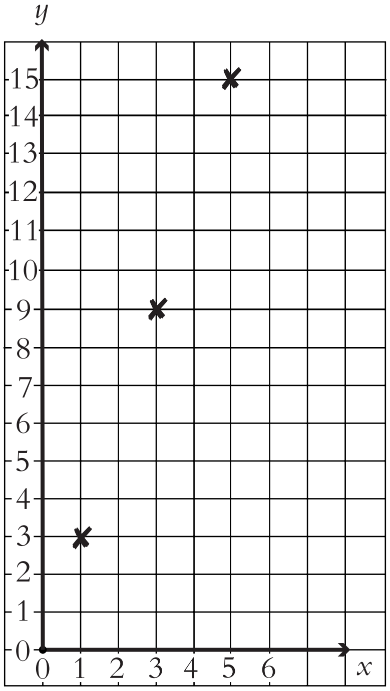
(b) Draw a straight line through these points.

(c) Write down the co-ordinates of two other points on this line.

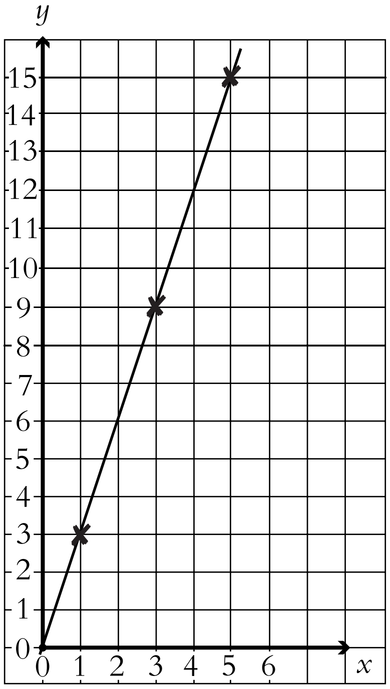
(d) Describe how the x and y co-ordinates are related.

Solution

(a) The points are plotted below:



(b) A line can then be drawn through these points:



(c) The points (2, 6) and (4, 12) also lie on the line (there are many others).

(d) The y co-ordinate is three times the x co-ordinate. This is written as $y = 3x$.

Skill Exercises: Plotting Points on a Straight Line

1. (a) Plot the points with co-ordinates (0, 4), (1, 5), (3, 7) and (5, 9).
   (b) Draw a straight line through the points.
   (c) Write down the co-ordinates of three other points that lie on this line.

2. (a) Plot the points with co-ordinates (0, 6), (2, 4), (3, 3) and (5, 1), and draw a straight line through them.

   (b) On the same graph as used for question 2(a), plot the points with co-ordinates (1, 8), (2, 7), (5, 4) and (7, 2), and draw a straight line through them.

   (c) Copy and complete the sentence:

   "These two lines are p__________."

3. (a) Plot the points with co-ordinates (2, 6), (3, 5), (4, 4) and (7, 1), and draw a straight line through them.

   (b) On the same set of axes, plot the points with co-ordinates (0, 1), (1, 2), (3, 4) and (5, 6), and draw a straight line through them.

   (c) Copy and complete this sentence:

   "These two lines are p__________."

4. (a) Plot the points with co-ordinates (1, 1), (2, 2), (4, 4) and (5, 5) and draw a straight line through them.

   (b) Write down the co-ordinates of two other points on the line.

   (c) Describe the relationship between the x and y co-ordinates.

5. The points (1, 3), (2, 4), (3, 5) and (5, 7) lie on a straight line.

   (a) Plot these points and draw the line.

   (b) Write down the co-ordinates of three other points on the line.

   (c) Describe the relationship between the x and y co-ordinates.

6. (a) Plot the points (0, 5), (2, 3), (4, 1) and (5, 0). Draw a straight line through them.

   (b) Write down the co-ordinates of two other points on the line.

   (c) The relationship between the x and y co-ordinates can be written as $x + y = \boxed{}$. What is the missing number?

7. (a) Plot the points with co-ordinates ($^-3$, $^-4$), ($^-1$, $^-2$), (1, 0), (4, 3).

   (b) Draw a straight line through these points.

   (c) Describe the relationship between the x and y co-ordinates.

8. The points with co-ordinates ($^-2$, $^-4$), (2, 4), (3, 6) and (4, 8) lie on a straight line.

   (a) Draw the line.

   (b) Describe the relationship between the x and y co-ordinates of points on the line.

9. The points with co-ordinates $(^-6, ^-3)$, $(^-1, 2)$, $(2, 5)$ and $(4, 7)$ lie on a straight line.

   (a) Draw the line.

   (b) Complete the missing numbers in the co-ordinates of the other points that lie on the line:

   $(^-7,$ ☐ $)$, $($☐$, ^-1)$, $(3,$ ☐ $)$, $($☐$, 4)$, $(100,$ ☐ $)$.

   (c) Describe the relationship between the x and y co-ordinates of the points on the line.

   (d) Will the point with co-ordinates $(25, 27)$ lie on the line? Give a reason for your answer.

10. Each set of points listed below lies on a straight line. Plot the points, draw the line, and complete the statement about the relationship between the x and y co-ordinates.

   (a) $(1, 6)$, $(3, 4)$, $(8, ^-1)$ $x + y =$ ☐

   (b) $(^-4, 2)$, $(^-1, 5)$, $(3, 9)$ $y = x +$ ☐

   (c) $(^-2, ^-8)$, $(0, 0)$, $(3, 12)$ $y =$ ☐ x

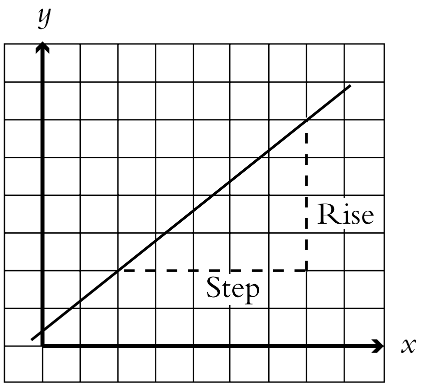
   (d) $(^-4, ^-6)$, $(^-1, ^-3)$, $(3, 1)$ $y = x -$ ☐

Drawing Graphs given their Equations

In this section we see how to plot a graph, given its equation. We also look at how steep it is and use the word gradient to describe this. There is a simple connection between the equation of a line and its gradient, which you will notice as you work through this section.

Gradient of a Line

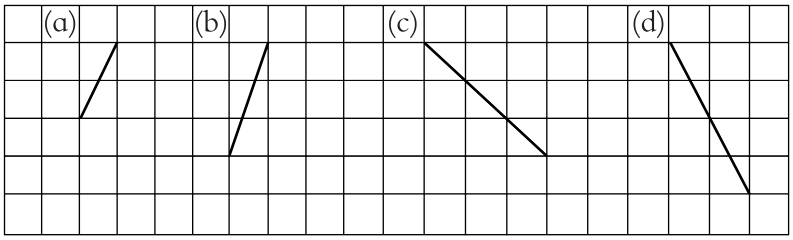
$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}}$$



You can draw any triangle using the sides to determine the rise and step, but the triangle must have one side horizontal and one side vertical.

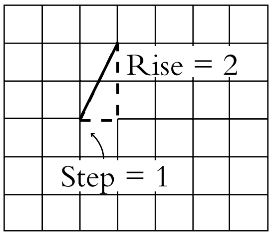
Example 1

Determine the gradient of each of the following lines:



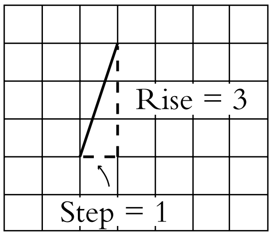
Solution

(a)



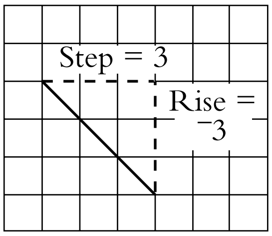
$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}} = \frac{2}{1} = 2$$

(b)



$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}} = \frac{3}{1} = 3$$

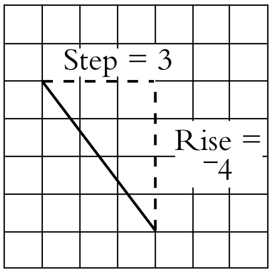
(c)



$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}} = \frac{(^-3)}{3} = {}^-1$$

Note that in (c) the rise is negative although the step is positive. The gradient of the line is negative.

(d)



$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}} = \frac{(^-4)}{3} = {}^-\frac{4}{3}$$

Note that in (d) again the rise is negative, and the step is positive. The gradient of the line is negative.

(In both (c) and (d) you will see that the lines slope in a different direction to the lines in (a) and (b), which have a positive gradient.)

Example 2

(a) Complete the table below for $y = 2x + 1$.

x	$^-2$	$^-1$	0	1	2
y					

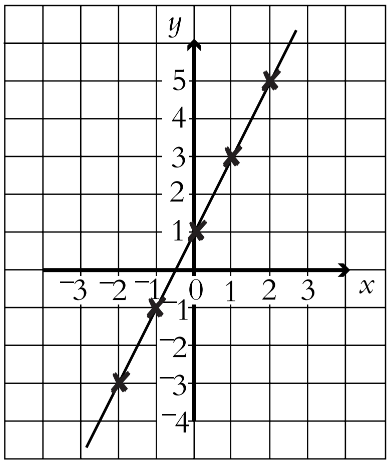
(b) Use the information in the table to plot the graph with equation $y = 2x + 1$.

Solution

(a)

x	$^-2$	$^-1$	0	1	2
y	$^-3$	$^-1$	1	3	5

(b) The points ($^-2$, $^-3$), ($^-1$, $^-1$), (0, 1), (1, 3) and (2, 5) can then be plotted, and a straight line drawn through these points.



Example 3

(a) Draw the graph of the line with the equation $y = x + 1$.

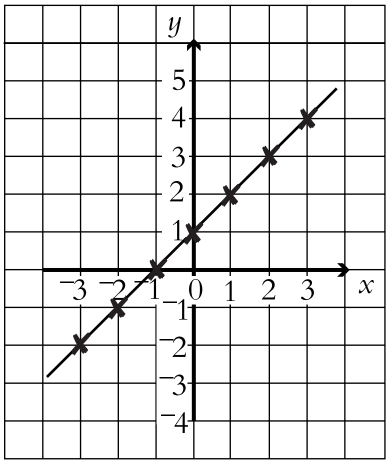
(b) What is the gradient of the line?

Solution

(a) The table shows how to calculate the co-ordinates of some points on the line.

x	$^-3$	$^-2$	$^-1$	0	1	2	3
y	$^-2$	$^-1$	0	1	2	3	4

The points with co-ordinates ($^-3, ^-2$), ($^-2, ^-1$), ($^-1, 0$), (0, 1), (1, 2), (2, 3) and (3, 4) can then be plotted and a line drawn as shown:

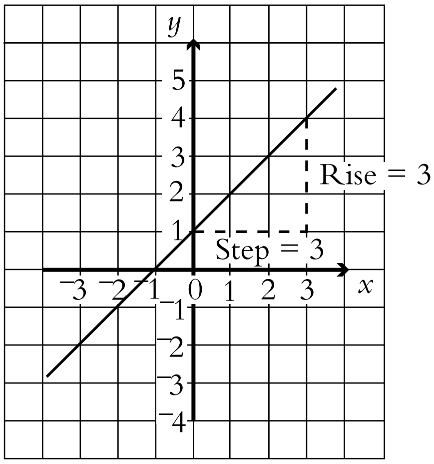


(b) To calculate the gradient of the line, draw a triangle under the line as shown in the diagram below. The triangle can be of any size, but must have one horizontal side and one vertical side.

$$\text{Gradient} = \frac{\text{Rise}}{\text{Step}}$$

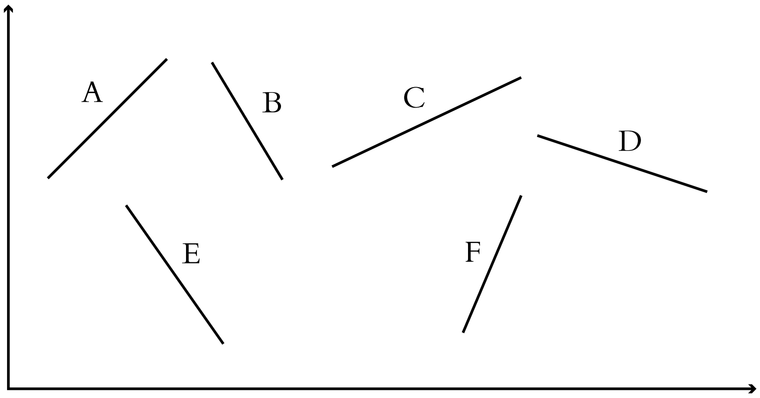
$$= \frac{3}{3}$$

$$= 1$$

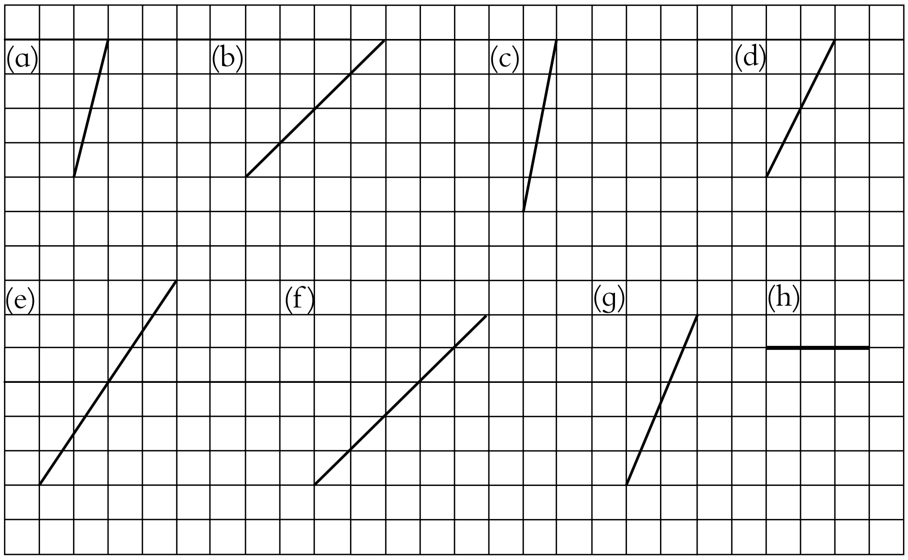


Skill Exercises: Drawing Graphs given their Equations

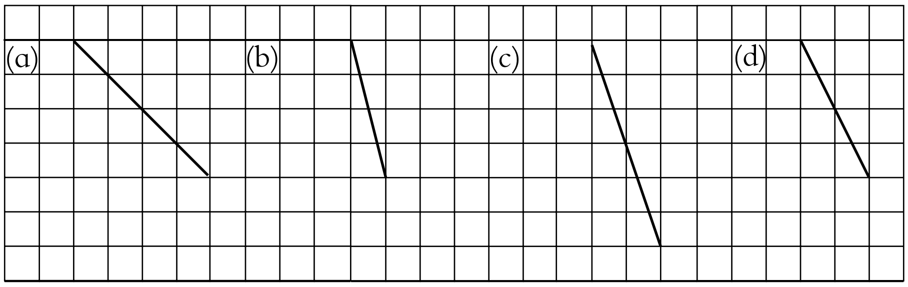
1. Which of the following lines have a positive gradient and which have a negative gradient?



2. Determine the gradient of each of the following lines:



3. Determine the gradient of each of the following lines:



4. (a) Copy and complete the following table for $y = x - 2$:

x	-3	-2	-1	0	1	2	3
y				-2			

(b) Draw the line with equation $y = x - 2$.

5. (a) Copy and complete the following table for $y = 2x + 3$:

x	-3	-2	-1	0	1	2	3
y							9

(b) Draw the line with equation $y = 2x + 3$.

6. (a) Draw the line with equation $y = 2x - 1$.

(b) Determine the gradient of this line.

7. (a) Draw the line with equation $y = \frac{1}{2}x + 2$.

(b) Determine the gradient of this line.

8. (a) Draw the lines $y = 3x + 1$ and $y = 4x - 5$.

(b) Determine the gradient of these lines.

9. Without drawing the lines, state the gradients of the lines with the following equations:

   (a) $y = 2x + 4$

   (b) $y = 3x - 9$

   (c) $y = 10x + 1$

   (d) $y = 5x + 3$

10. (a) Draw the lines and equations $y = 2x + 1$ and $y = 3x - 2$.

    (b) Write down the co-ordinates of the point where these two lines cross.

11. Determine the co-ordinates of the point where the lines $y = x + 3$ and $y = 7 - x$ cross.

12. (a) Draw the line with equation $y = 6 - 2x$.

    (b) Explain why the gradient of this line is $^-2$.

13. (a) Explain why the lines with equations $y = 2 - 2x$ and $y = 5 - 2x$ are parallel.

    (b) Write down the equation of another line that would be parallel to these lines.

    (c) Draw all three lines.

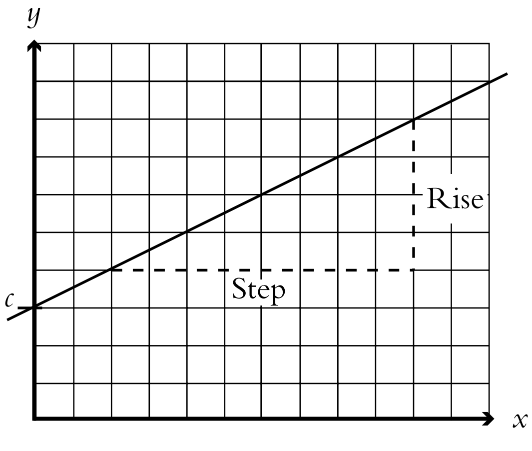
Finding the Equation of a Straight Line

The equation of a straight line contains information about the gradient of the line and the point where it crosses the y-axis.

The y intercept is c: that is the point where the line crosses the y-axis.

The gradient is m, where

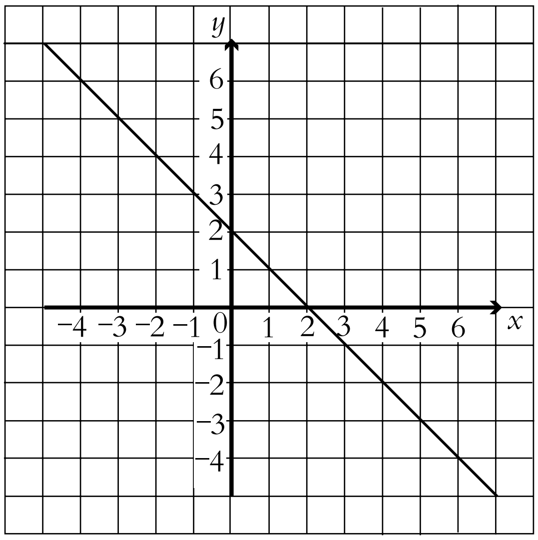
$$m = \frac{\text{Rise}}{\text{Step}}$$



The equation of a straight line is $y = mx + c$.

Example 1

(a) Determine the equation of the line shown below:

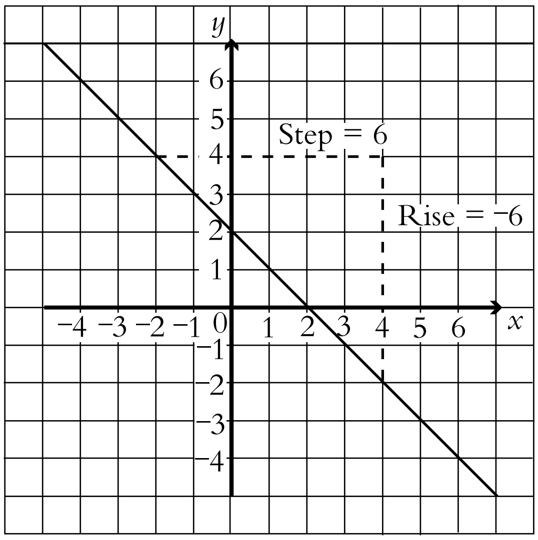


Solution

On the graph below, first note that the intercept is 2, therefore we write $c = 2$.

Next calculate the gradient of the line.

Note that when the step is 6 the rise is $^{-}6$. The line is going down as you move from left to right.



$$\text{Gradient} = \frac{^{-}6}{6}$$

$$= {}^{-}1$$

$$\text{Therefore } m = {}^{-}1$$

The equation of a straight line is $y = mx + c$. With $m = {}^{-}1$ and $c = 2$, we have:

$$y = {}^{-}x + 2$$

or

$$y = 2 - x$$

Reminder: Recall that $^{-}1 \times x = {}^{-}1x$ is written as ^{-}x for speed and convenience.

Skill Exercises: Finding the Equation of a Straight Line

1. (a) Draw the line with equation $y = 2x + 3$.

   (b) Determine the gradient of the line.

   (c) What is the y intercept of this line?

2. (a) Draw the lines with equations $y = x$, $y = {}^-x$, $y = 2x$ and $y = {}^-3x$.

   (b) Determine the gradient of each line.

   (c) What is the y intercept of each line?

3. The points with co-ordinates $({}^-2, 3)$, $(0, 5)$ and $(3, 8)$ lie on a straight line.

   (a) Plot the points and draw the line.

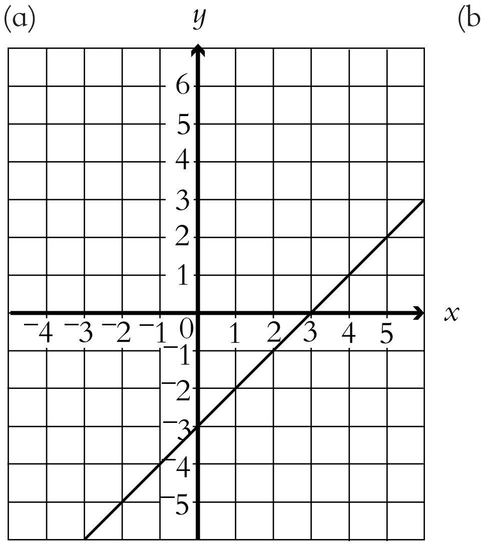
   (b) Determine the gradient of the line.

   (c) What is the y intercept of this line?

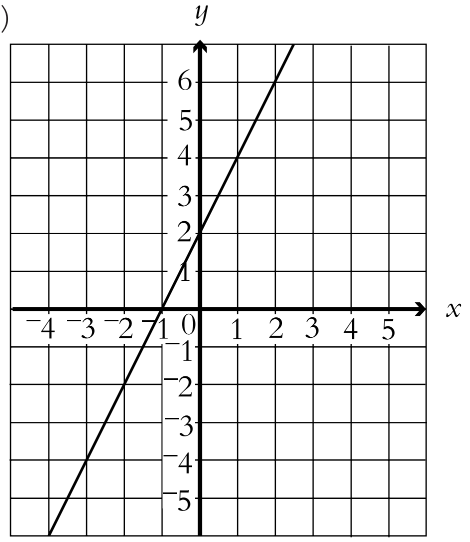
   (d) Write down the equation of the line.

4. Determine the equation of each of the lines shown below:

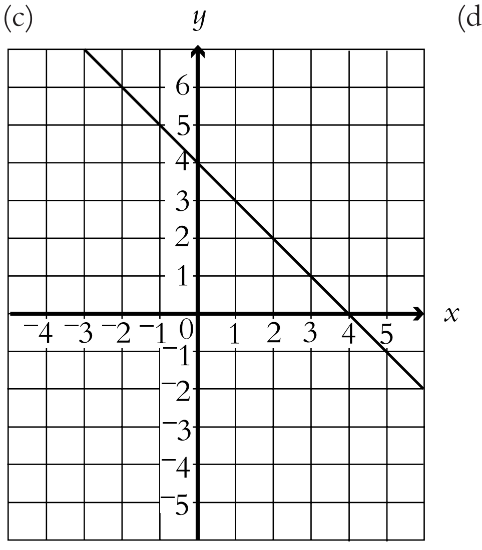
(a)



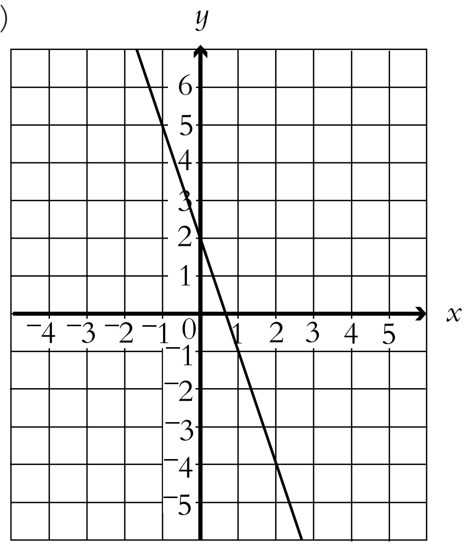
(b)



(c)



(d)



5. Copy and complete the following table:

Equation	Gradient	Intercept
$y = 2x + 7$		
	8	$^-2$
$y = 8 - 3x$		
	7	$^-5$
	$^-3$	2
	$^-5$	$^-2$

6. (a) Draw the lines with equations $y = x + 1$, $y = 1 - x$, $y = 2x + 1$ and $y = 3x + 1$ on the same set of axes.

   (b) Explain why these lines all pass through the same point on the y-axis.

7. The points with co-ordinates $(^-2, ^-6)$, $(0, 0)$ and $(3, 9)$ all lie on a straight line.

   (a) What is the gradient of the line?

   (b) What is the intercept of the line?

   (c) What is the equation of the line?

8. Draw lines which have:

   (a) Gradient 2 and intercept 3.

   (b) Gradient $\frac{1}{2}$ and intercept 1.

   (c) Gradient $^-4$ and intercept 7.

Section 9.2 Expanding And Factorising Expressions

Expanding Expressions with Two Brackets

When two brackets are multiplied together, for example,

$$(x + 2)(x + 3)$$

every term in the first bracket must be multiplied by every term in the second bracket.

Example 1

Use a table to determine

$$(x + 2)(x + 3)$$

Solution

$(x + 3)$

$(x + 2)$

$\times$	x	3
x	x^2	$3x$
2	$2x$	6

The multiplication table is formed using the two brackets.

The contents of the table give the expansion.

$$\begin{array}{rclcrl}
(x+2)(x+3) & = & x^2+3x+2x+6 & \text{or} & & x^2+3x \\
 & = & x^2+5x+6 & & & \underline{+2x+6} \\
 & & & & = & \underline{x^2+5x+6}
\end{array}$$

Example 2

Use a table to determine

$$(x - 6)(x + 2)$$

Solution

$(x + 2)$

$(x - 6)$

$\times$	x	2
x	x^2	$2x$
$^{-}6$	$^{-}6x$	$^{-}12$

Therefore,

$$\begin{array}{rclcrl}
(x-6)(x+2) & = & x^2+2x-6x-12 & \text{or} & & x^2+2x \\
 & = & x^2-4x-12 & & & \underline{-6x-12} \\
 & & & & = & \underline{x^2-4x-12}
\end{array}$$

An alternative method for expanding two brackets is shown in the next example.

Example 3

Calculate $(x + 2)(x - 7)$

Solution

$$\begin{array}{rcllr} (x + 2)(x - 7) & = & x(x - 7) + 2(x - 7) & & \\ & = & x^2 - 7x + 2x - 14 & \text{or} & x^2 - 7x \\ & = & x^2 - 5x - 14 & & \underline{+2x - 14} \\ & & & = & \underline{x^2 - 5x - 14} \end{array}$$

Note how each term in the first bracket multiplies each term of the second bracket.

Skill Exercises: Expanding Expressions with Two Brackets

1. Copy and complete the following tables and write down each of the expansions:

(a)

×	x	5
x		
4		

$(x + 4)(x + 5)$

(b)

×	x	$^{-}7$
x		
4		

$(x + 4)(x - 7)$

(c)

×	x	4
x		
$^{-}1$		

$(x - 1)(x + 4)$

(d)

×	x	$^{-}5$
x		
$^{-}2$		

$(x - 2)(x - 5)$

2. Expand:

(a) $(x + 3)(x + 4)$
(b) $(x - 2)(x + 5)$
(c) $(x - 5)(x - 1)$
(d) $(x + 7)(x - 3)$
(e) $(x + 2)(x - 3)$
(f) $(x + 4)(x - 1)$

3. Expand:

(a) $(x - 1)(x + 1)$
(b) $(x + 2)(x - 2)$
(c) $(x - 5)(x + 5)$
(d) $(x - 7)(x + 7)$
(e) How are the answers to this question different from the others you have done?

4. Explain what is wrong with this statement:

   $(x + 5)^2 = x^2 + 25$

5. Expand:

   (a) $(x + 1)^2$ (b) $(x - 1)^2$

   (c) $(x + 3)^2$ (d) $(x - 5)^2$

6. (a) Copy and complete this table:

×	x	6
$2x$		
1		

   (b) What is the expansion of $(2x + 1)(x + 6)$?

7. Expand:

   (a) $(2x + 1)(2x + 4)$ (b) $(3x + 1)(4x + 1)$

   (c) $(2x - 1)(3x + 2)$ (d) $(4x - 1)(5x + 1)$

   (e) $(2x + 1)^2$ (f) $(4x - 3)^2$

8. Write out the following expansions, filling the missing terms:

   (a) $(x + 7)(x + 6) = x^2 + ? + 42$

   (b) $(x + 6)^2 = x^2 + ? + 36$

   (c) $(x - 2)(x - 5) = x^2 + ? + 10$

   (d) $(x - 1)(2x + 1) = 2x^2 - x - ?$

   (e) $(x + 3)(2x + 1) = ? + 7x + 3$

   (f) $(x - 7)^2 = x^2 - ? + 49$

9. Explain what is wrong with this statement:

   $(x + 4)(x - 5) = x^2 - 20$

10. Write out the following expansions, filling in the missing terms:

   (a) $(x + ?)(x - 1) = x^2 + x - 2$

   (b) $(x + 4)(x - ?) = x^2 - 2x - 24$

   (c) $(2x + 3)(x + ?) = 2x^2 + 9x + ?$

   (d) $(x - ?)(x + 5) = x^2 - 2x - ?$

   (e) $(x + ?)(x + ?) = x^2 + 4x + 4$

   (f) $(x + ?)(x + ?) = x^2 + 6x + 8$

Factorising Expressions

The process of factorising involves introducing brackets into expressions.

Example 1

Factorise:

(a) $8x + 12$ (b) $35x + 28$

Solution

(a) Both terms are multiples of 4

$\therefore\ 8x + 12 = 4(2x + 3)$

(b) Both terms are multiples of 7

$\therefore\ 35x + 28 = 7(5x + 4)$

Results like this can be checked by multiplying out the bracket to get back to the original expression.

Example 2

Factorise:

(a) $x^2 + 2x$ (b) $3x^2 - 9x$ (c) $x^3 - x^2$

Solution

(a) Both terms are multiples of x

$\therefore\ x^2 + 2x = x(x + 2)$

(b) Both terms are multiples of x and 3

$\therefore\ 3x^2 - 9x = 3x(x - 3)$

(c) Both terms are multiples of x^2

$\therefore\ x^3 - x^2 = x^2(x - 1)$

Sometimes it is possible to factorise in stages. For example, in part (b), you could have worked like this:

$$\begin{aligned} 3x^2 - 9x &= 3(x^2 - 3x) \\ &= 3x(x - 3) \end{aligned}$$

Example 3

Factorise:

(a) $x^2 + 9x + 18$ (b) $x^2 + 2x - 15$ (c) $x^2 - 7x + 12$

Solution

(a) This expression will need to be factorised into two brackets:

$x^2 + 9x + 18 = (x \quad)(x \quad)$

As the expression begins x^2, both brackets must begin with x. The two numbers to go in the brackets must multiply together to give 18 and add to give 9. Therefore, they must be 3 and 6, giving,

$x^2 + 9x + 18 = (x + 3)(x + 6)$

You can check this result by multiplying out the brackets.

(b) We note first that two brackets are needed and that both must contain an x, as shown:

$x^2 + 2x - 15 = (x \quad)(x \quad)$

The two other numbers are needed which, when multiplied give $^{-}15$ and when added give 2. In this case, these are $^{-}3$ and 5. Therefore the factorisation is,

$x^2 + 2x - 15 = (x - 3)(x + 5)$

Check this result by multiplying out the brackets.

(c) Again, we begin by noting that,

$x^2 - 7x + 12 = (x \quad)(x \quad)$

We require two numbers which, when multiplied give 12 and when added give $^{-}7$. In this case, these numbers are $^{-}3$ and $^{-}4$.

$x^2 - 7x + 12 = (x - 3)(x - 4)$

Skill Exercises: Factorising Expressions

1. Factorise:

(a) $4x - 2$ (b) $6x - 12$ (c) $5x - 20$

(d) $4x + 32$ (e) $6x - 8$ (f) $8 - 12x$

(g) $21x - 14$ (h) $15x + 20$ (i) $30 - 10x$

2. Factorise:

(a) $x^2 + 4x$ (b) $x^2 - 3x$ (c) $4x - x^2$

(d) $6x^2 + 8x$ (e) $9x^2 + 15x$ (f) $7x^2 - 21x$

(g) $28x - 35x^2$ (h) $6x^2 - 14x$ (i) $5x^2 - 3x$

3. Factorise:

   (a) $x^3 + x^2$
   (b) $2x^2 - x^3$
   (c) $4x^3 - 2x^2$
   (d) $8x^3 + 4x^2$
   (e) $16x^2 - 36x^3$
   (f) $4x^3 + 22x^2$
   (g) $16x^2 - 6x^3$
   (h) $14x^3 + 21x^2$
   (i) $28x^3 - 49x^2$

4. (a) Expand $(x + 5)(x - 5)$

   (b) Factorise $x^2 - 25$

   (c) Factorise each of the following:

      (i) $x^2 - 49$

      (ii) $x^2 - 64$

      (iii) $x^2 - 100$

      (iv) $x^2 - a^2$

      (v) $x^2 - 4b^2$

5. Factorise:

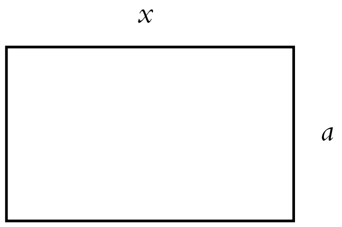
   (a) $x^2 + 7x + 12$
   (b) $x^2 + 8x + 7$
   (c) $x^2 + 11x + 18$
   (d) $x^2 + 12x + 27$
   (e) $x^2 + 17x + 70$
   (f) $x^2 + 6x + 8$
   (g) $x^2 + 16x + 28$
   (h) $x^2 + 18x + 77$
   (i) $x^2 + 16x + 63$

6. Factorise:

   (a) $x^2 + x - 2$
   (b) $x^2 + x - 20$
   (c) $x^2 - x - 12$
   (d) $x^2 - 13x + 36$
   (e) $x^2 - 10x + 16$
   (f) $x^2 + x - 42$
   (g) $x^2 + 13x - 30$
   (h) $x^2 - 17x + 72$
   (i) $x^2 - 2x - 99$

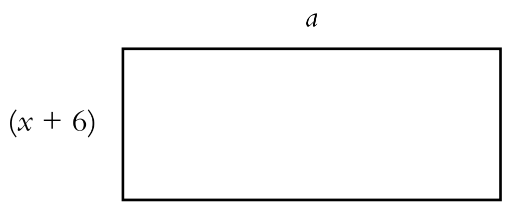
7. The area of the rectangle shown is $x^2 - 5x$.

   Express a in terms of x.



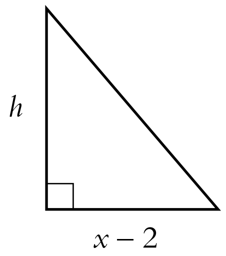
8. The area of the rectangle shown is $x^2 + 11x + 30$.

   Express a in terms of x.



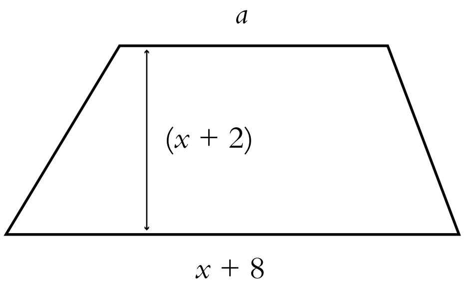
9. The area of the triangle shown is $\frac{1}{2}x^2 + \frac{3}{2}x - 5$.

   Express h in terms of x.



10. The area of the trapezium shown is $\frac{1}{2}x^2 + 10x + 18$.

    Calculate a.



Unit 10: STATISTICS AND PROBABILITY – PART 2

In this unit you will be:

10.1 Displaying Data

- Displaying Data on a Line Graph.
- Displaying Data on a Stem and Leaf Plot.

10.2 Analysing Data

- Calculating the Mean, Mode, Median and Range.

10.3 Reporting Statistical Findings

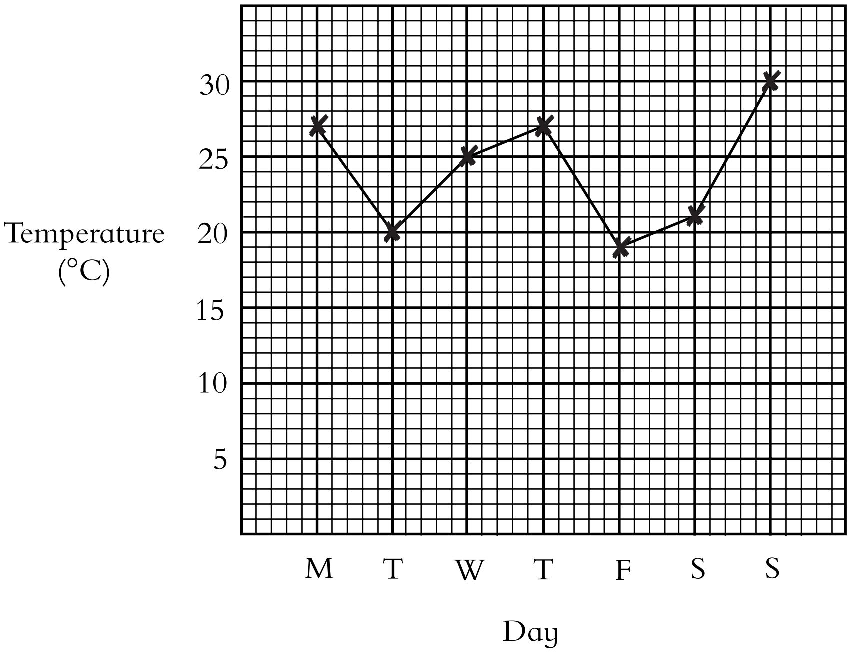
Section 10.1 Displaying Data

Displaying Data on a Line Graph

A line graph is drawn by plotting data points and joining them with straight lines. It is really only the actual data points that count, but by drawing the lines you get a better impression of the trend given by the data. This method of representation is particularly useful when illustrating trends over time.

Example 1

Tasi recorded the temperature at 6 pm each day for a week. His records are shown on this line graph:



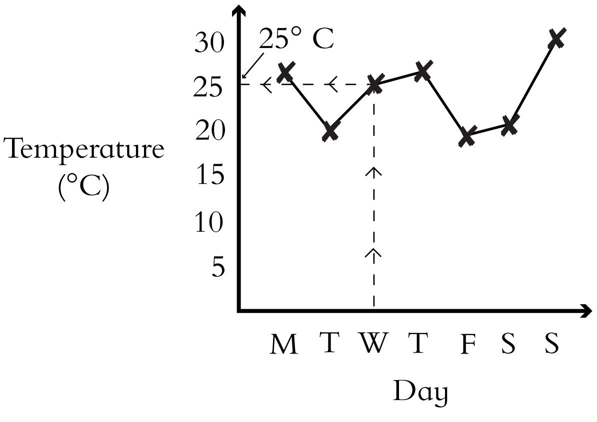
(a) What was the temperature on Wednesday?

(b) What was the lowest temperature recorded?

(c) What was the highest temperature recorded?

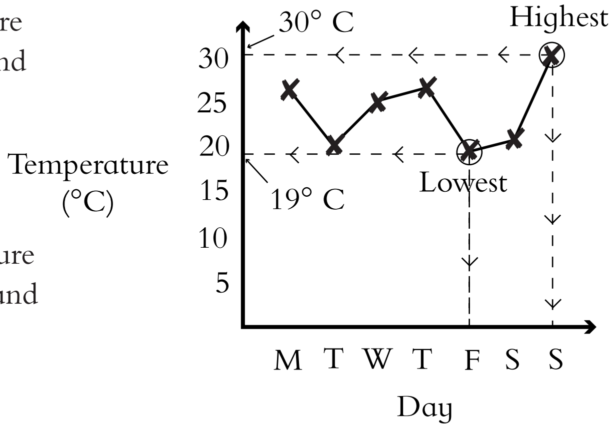
Solution

(a) For Wednesday the temperature can be read as 25°C.



(b) The lowest temperature occurred on Friday and was 19°C.

(c) The highest temperature occurred on Sunday and was 30°C.



Example 2

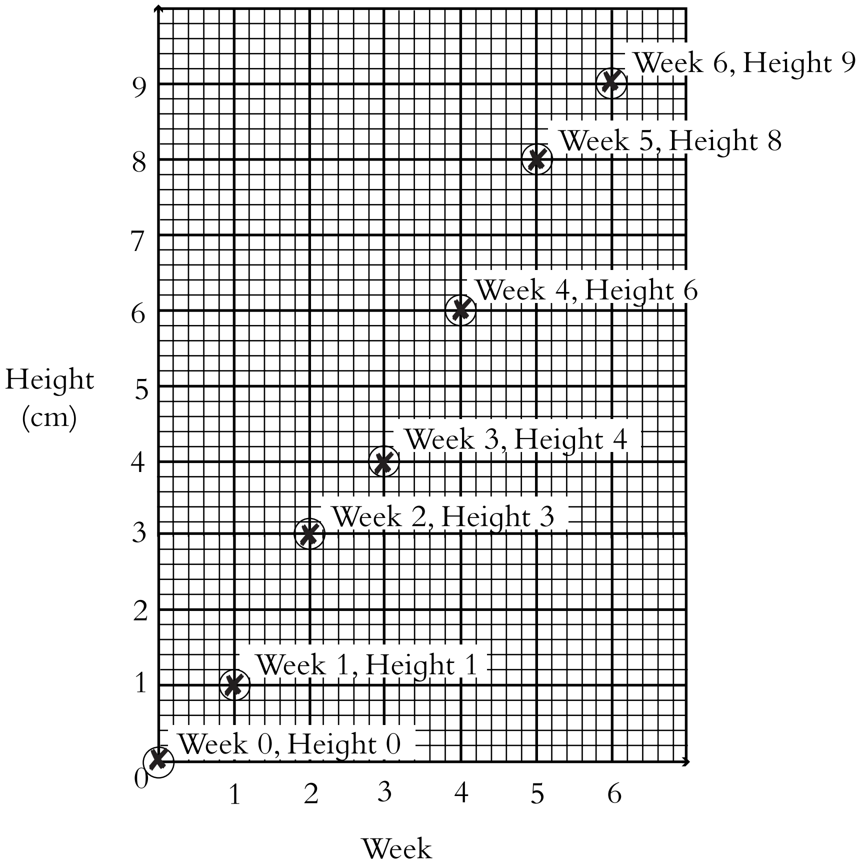
As part of a science project Lani records the height of a plant every week. His results are in this table:

Week	0	1	2	3	4	5	6
Height (cm)	0	1	3	4	6	8	9

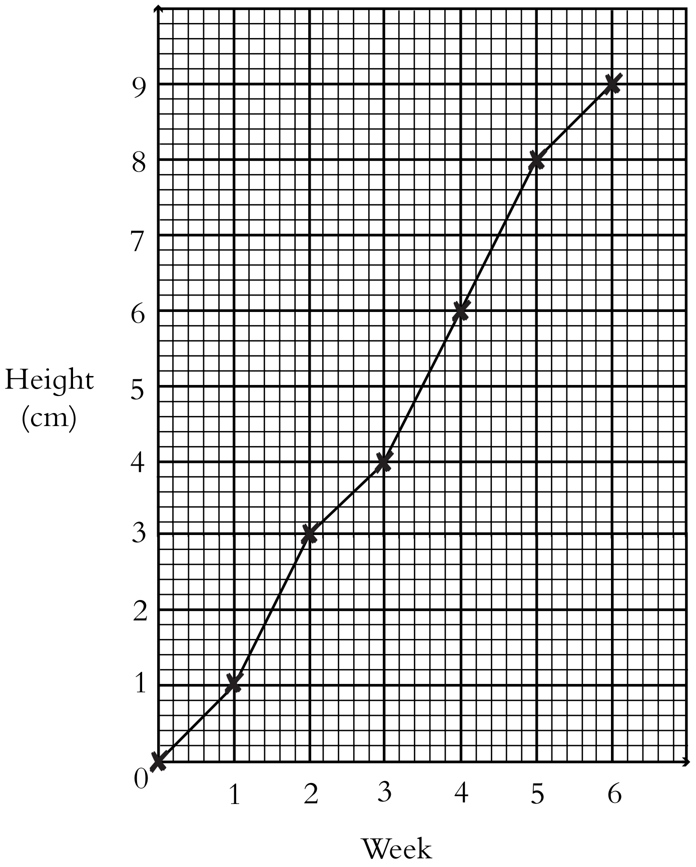
Draw a line graph.

Solution

First draw a suitable set of axes. Then plot a point for each measurement as shown below.

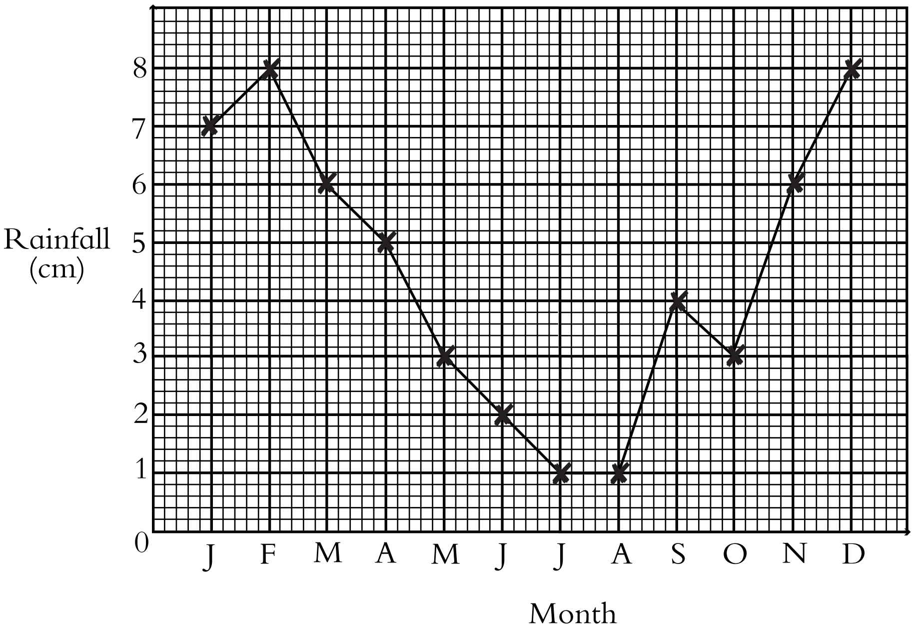


The points can be joined with straight lines as shown in this graph.



Skill Exercises: Displaying Data on a Line Graph

1. The line graph shows the monthly rainfall for a town.



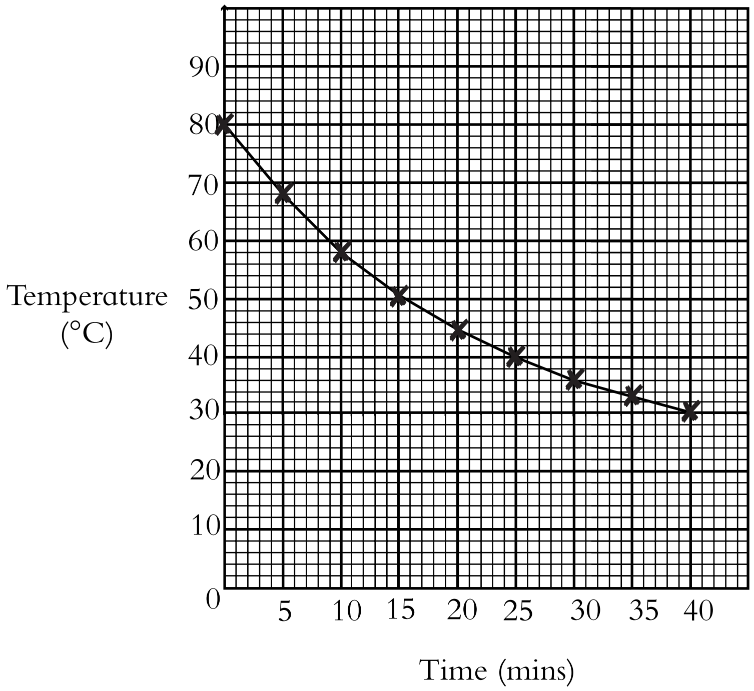
(a) How much rain was there in September?

(b) In which month was the rainfall 5 cm?

(c) Which months were the wettest?

(d) Which months were the driest?

2. A cup was filled with hot water and the temperature was recorded every five minutes. The graph below shows the results.



(a) What was the temperature after 25 minutes?

(b) What was the temperature at the start of the experiment?

(c) When was the temperature 45°C?

(d) How long did it take for the temperature to drop from 68°C to 36°C?

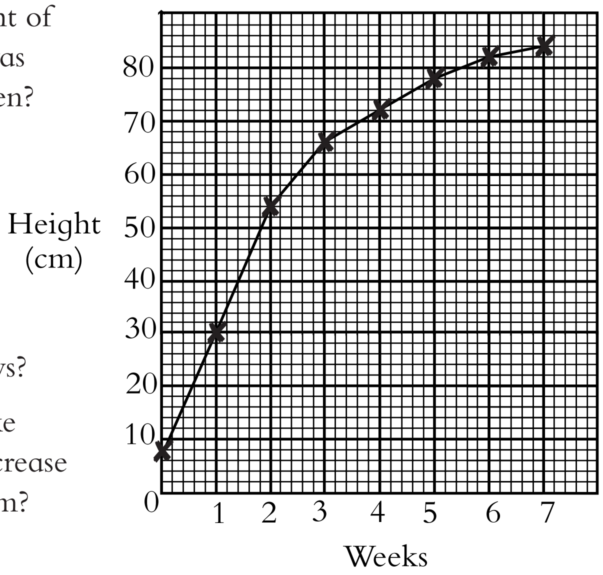
3. The following graph shows how the height of a sunflower plant changed since it was planted in a garden.

(a) What was the height of the plant when it was planted in the garden?

(b) How much did the plant grow in the first week?

(c) What is the greatest height that the graph shows?

(d) How long did it take for the height to increase from 54 cm to 78 cm?



4. Paulo recorded the temperature outside his house at 8.00 am every day. His results are in this table.

Day	M	T	W	T	F	S	S
Temperature (°C)	28	25	24	26	27	25	23

Draw a line graph for this data.

5. Karen counted the number of cars that drove past her while she was waiting at the bus stop each morning on her way to school.

Day	M	T	W	T	F	S
Number of Cars	18	12	22	36	4	10

Draw a line graph for this data.

6. Ana recorded the time it took to walk to school every day for a week.

Day	M	T	W	T	F
Time taken (mins)	8	9	15	12	7

Draw a line graph for this data.

7. Sitivi is training to run a marathon. Each week he recorded the time it took to run five miles.

Week	0	1	2	3	4	5	6
Time (mins)	52	50	46	44	40	36	34

Draw a line graph for this data.

Displaying Data on a Stem and Leaf Plot

There are many ways of representing data. For example, you are probably familiar with histograms and bar charts but there is another very simple way which quickly gives an overall view of the general characteristics of the data. This is called a Stem and Leaf Plot. The following example illustrates how it works.

Example 1

The marks gained out of 50 by 15 pupils in a Biology test are given below.

27	36	24	17	35	18	23	25
34	25	41	18	22	24	42	

Solution

Form a Stem and Leaf Plot by recording the marks with the tens as the 'stem' and the units as the 'leaf', as shown opposite.

Stem	Leaf
0	
1	7 8 8
2	7 4 3 5 5 2 4
3	6 5 4
4	1 2

The leaf part is then reordered to give a final plot as shown.

Stem	Leaf
0	
1	7 8 8
2	2 3 4 4 5 5 7
3	4 5 6
4	1 2

This gives at a glance both an impression of the spread of the numbers and an indication of the average.

Skill Exercises: Displaying Data on a Stem and Leaf Plot

1. A class of pupils take a test. Their scores are listed below:

17	23	46	31	17	19	26	31	42	5
21	32	36	37	37	38	41	40	19	12
7	48	29	39	42	38	41	32	36	35

Draw a stem and leaf diagram for this data.

2. Form a Stem and Leaf Plot for the following data:

21	7	9	22	17	15	31	5	17	22	19	18	23
10	17	18	21	5	9	16	22	17	19	21	20	

3. The ages of drivers involved in fatal road accidents in Australia during one week are given below:

17	82	40	48	21	35	23	24	18	57	62	45
20	21	33	27	24	37	58	69	65	19	15	21
28	71	43	31	73	26	18	21	34	35	51	63
23	65	22	45	23	27	18	19	32	25	61	36

Illustrate this data using a Stem and Leaf Plot.

Section 10.2 Analysing Data

Calculating the Mean, Mode, Median and Range

The mean, median and mode are types of average. The range gives a measure of the spread of a set of data.

Definition	Example
$\text{Mean} = \dfrac{\text{sum of all data}}{\text{number of values}}$	For 1, 2, 2, 3, 4 $\text{Mean} = \dfrac{1+2+2+3+4}{5}$ $= \dfrac{12}{5}$ $= 2.4$
Mode = most common value	For 1, 2, 2, 3, 4 Mode = 2 For 1, 2, 2, 3, 4, 4, 5 Mode = 2 and 4
Median = middle value when data is arranged in order	For $1, 2, \enclose{circle}{2,} 3, 4$ Median = 2 For $1, 2, \enclose{circle}{2, 3,} 4, 4$ $\text{Median} = \dfrac{2+3}{2}$ $= 2.5$
Range = largest value – smallest value	For 1, 2, 2, 3, 4 Range = 4 – 1 = 3

The mean, median, mode and range can also be calculated when the data is presented in the form of a frequency table.

Example 1

For the data presented in the table opposite, calculate:

(a) The mode.
(b) The range.
(c) The median.
(d) The mean.

Score	Frequency
0	2
1	6
2	12
3	4
4	1

Solution

(a) The mode is the most common score. In this case,

mode = 2

(b) Largest score = 4, smallest score = 0,

range = 4 – 0
= 4

(c) The median is the middle value. As there are 25 scores, the middle value is the 13th score (12 above and 12 below).

When in order:

the first 2 values are 0
the next 6 values are 1
therefore the 3rd to 8th values are 1.

The next 12 values are 2,
therefore the 9th to 20th values are 2.
The 13th value is in this group
therefore the 13th value is 2.

∴ the median = 2.

(d) To calculate the mean, complete a table like the one below:

Score	Frequency	Score × Frequency
0	2	0 × 2 = 0
1	6	1 × 6 = 6
2	12	2 × 12 = 24
3	4	3 × 4 = 12
4	1	4 × 1 = 4
Totals	25	46

Twelve pupils scored 2, a total of 24

Total number of pupils

Total of the scores

$$\text{Mean} = \frac{46}{25}$$

$$= 1.84$$

Example 2

Calculate the mean and median for the data in the table opposite:

Price ($)	Frequency
30	1
31	3
32	4
33	8
34	2
35	2

Solution

(a) To calculate the mean, construct a table like the one below:

Price ($)	Frequency	Price × Frequency
30	1	$30 \times 1 = 30$
31	3	$31 \times 3 = 93$
32	4	$32 \times 4 = 128$
33	8	$33 \times 8 = 264$
34	2	$34 \times 2 = 68$
35	2	$35 \times 2 = 70$
Totals	20	653

$$\text{Mean} = \frac{653}{20}$$

$$= \$32.65$$

As there are 20 values, the median will be between the 10th and 11th values.

When in order:

1st value is $30
Next 3 values are $31
    Therefore the 2nd to 4th values are $31
Next 4 values are $32
    Therefore the 5th to 8th values are $32
Next 8 values are $33
    Therefore 9th to 17th values are $33
Therefore both the 10th and 11th values are $33

Median = $33

Note: If the 10th and 11th values had been different from one another, we would have used a value halfway between them (the mean of the two numbers).

The symbol Σ (Greek 'sigma') means 'the sum of' or 'the total of'.

Therefore

$$\text{mean} = \frac{\Sigma \text{ frequency} \times \text{value}}{\Sigma \text{ frequency}}$$

Sometimes we use f to stand for frequency, x for the values and $\bar{x}$ for the mean. Therefore

$$\bar{x} = \frac{\Sigma f x}{\Sigma f}$$

Skill Exercises: Calculating Mean, Median, Mode and Range

1. Calculate the mean, median, mode and range for each set of data below:

   (a) 3 6 3 7 4 3 9

   (b) 11 10 12 12 9 10 14 12 9

   (c) 6 9 10 7 8 5

   (d) 2 9 7 3 5 5 6 5 4 9

2 (a) Copy and complete the table below:

Score	Frequency	Score × Frequency
0	2	
1	6	
2	8	
3	3	
4	0	
5	1	
Totals		

(b) Calculate the mean score.

3. The number of goals scored by a soccer team in each match of a season is listed below:

6	2	2	3	1	0	0	1	2	3
3	5	4	2	0	1	1	0	2	1
1	1	1	2	1	1	0	2	3	4

(a) Copy and complete the table below:

No. of Goals	Tally	Frequency	No. of Goals × Frequency
0			
1			
2			
3			
4			
5			
6			

(b) Calculate the mean.

(c) Calculate the median.

(d) What is the mode?

(e) What is the range?

4. The price of a litre of petrol at some garages was recorded and the results are given in the table below:

Price	Frequency
\$1.74	1
\$1.75	2
\$1.76	8
\$1.77	10
\$1.78	2
\$1.79	1
\$1.80	1

Calculate the mean, median and mode of these data.

5. A class collected data on the number of children in their families, and this information is listed below:

2	1	3	4	2	5	3	1	2	1
1	1	2	3	2	2	2	3	4	2
1	1	1	2	3	2	1	1	2	3

   (a) Calculate the mean number of children per family.

   (b) Calculate the median number of children per family.

   (c) Why are there no 'zeros'?

6. Professor Baker keeps a record of his golf scores, as shown in the table below:

Golf Score	Frequency
70	3
71	4
72	4
73	4
74	3
75	2

   Calculate his mean score.

7. A class collected data on their shoe sizes and presented it in the table below:

Shoe Size	Frequency
3	2
4	7
5	6
6	5
7	3
8	2

   (a) Calculate the mean, median and mode for the data.

   (b) Which of the three types of average is the most useful to a shoe shop manager ordering stock?

8. Tania sells vacuum cleaners and the table shows how many she sells each day in a 25-day period.

No. Sold per Day	Frequency
0	2
1	6
2	6
3	2
4	8
5	1

(a) Calculate the mean, median and mode for the data.

(b) Which of the averages gives the best impression of her sales figures?

9. Classes 8A and 8B have a sponsored spelling competition. The tables below give the number of correct spellings for both classes.

Class A

No. of Correct Spellings	Frequency
1	0
2	1
3	1
4	1
5	1
6	6
7	5
8	6
9	1
10	3

Class B

No. of Correct Spellings	Frequency
1	0
2	1
3	1
4	5
5	2
6	3
7	1
8	2
9	4
10	6

(a) Calculate the mean, median and mode for each class.

(b) Which average makes class A appear to be better at spelling?

(c) Which average makes class B appear to be better at spelling?

10. Paulo and Tavita play golf. The scores for their last 20 matches are given below:

Score	67	68	69	70	71	72	73	74	75	76	77	78
Paulo's Frequency	1	1	1	1	2	5	1	2	3	1	1	1
Tavita's Frequency	0	0	3	6	2	0	0	0	0	2	4	3

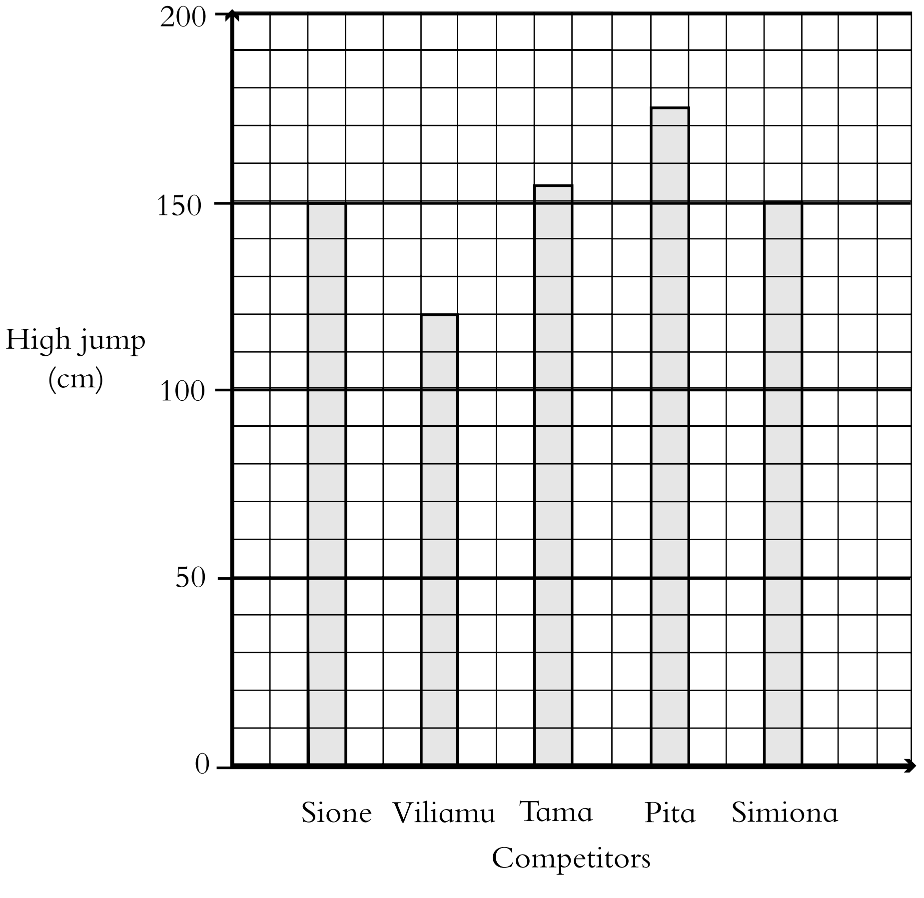
Produce arguments, supported by an average for each player, to show that each could be considered the better player.

Section 10.3 Reporting Statistical Findings

After classifying data and presenting it in a graph, it is easier to make sense of the data and understand what it is telling us.

Skill Exercises: Statistical Findings

1. The high jump data for a school is shown in the diagram below.



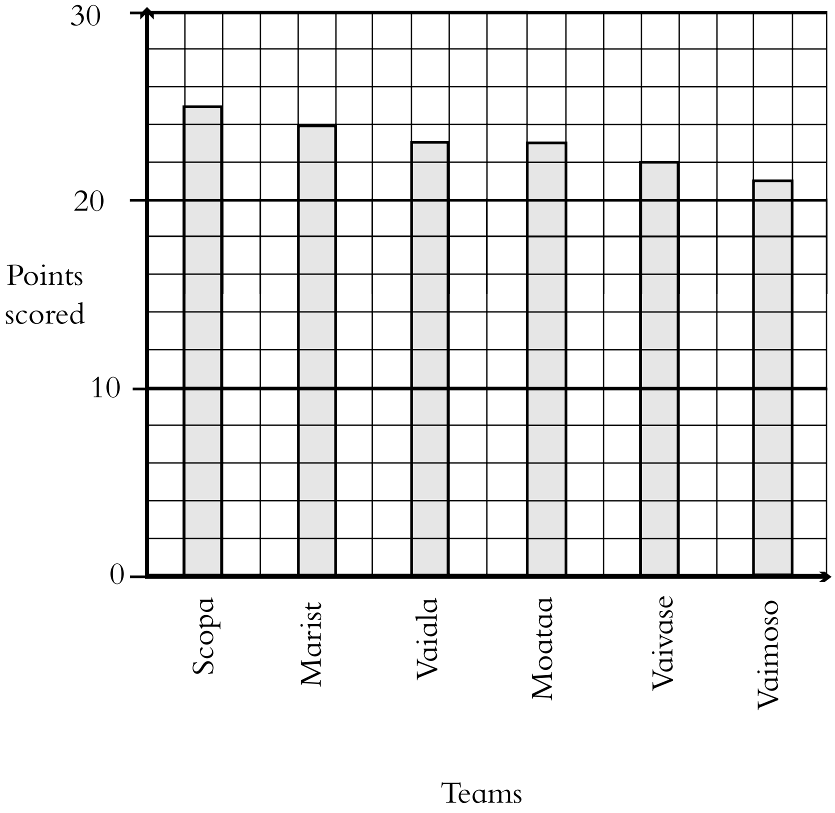
(a) Who won the competition, and what was the height of the jump?

(b) Where did Tama come in the competition?

(c) Which two competitors finished the competition with jumps of the same height?

(d) How much higher did the winner jump than the competitor who came second?

2. The diagram shows the number of points scored by the bottom six football teams, halfway through the season.



(a) What is the difference in points between the highest and the lowest of these teams?

(b) Which two teams have scored an equal number of points?

(c) Which team is two points ahead of Vaivase?

3. The average temperature in a number of cities in January is shown on the diagram.

   Use the information in the diagram to answer the following questions:

   (a) Which of these cities was the (i) warmest, (ii) coldest?

   (b) What was the difference in temperature between the warmest and the coldest cities?

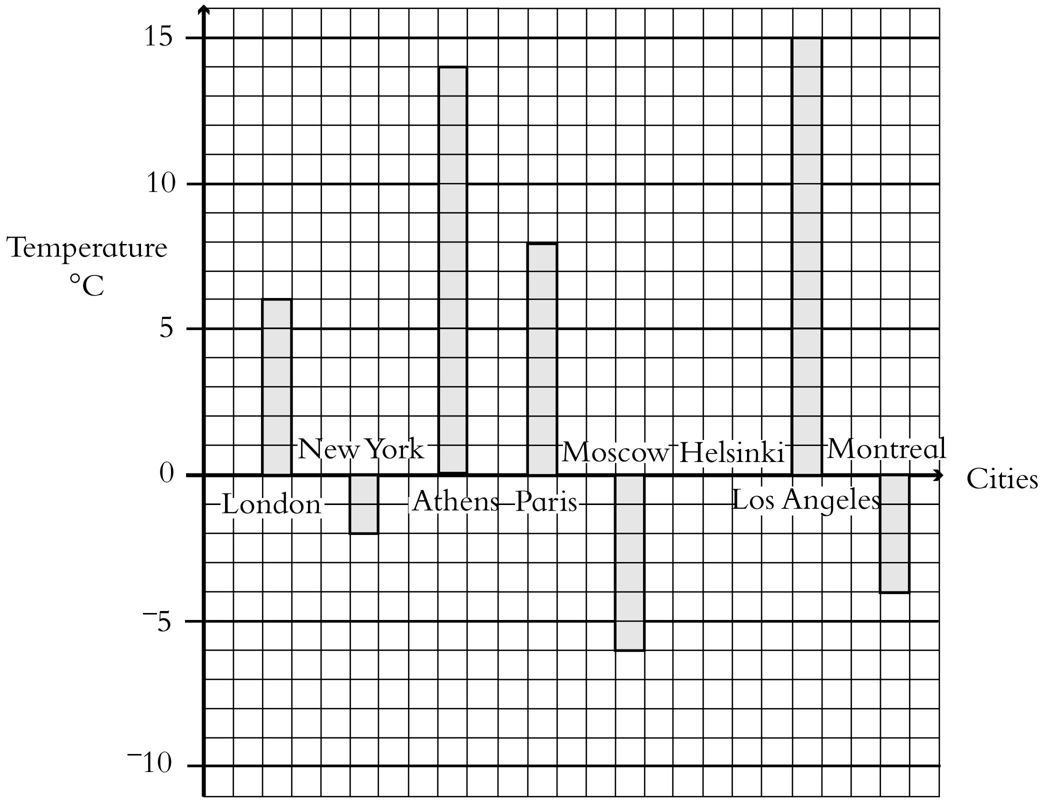
   (c) Which city was 4°C warmer than Moscow?

   (d) Which city was 2°C colder than Paris?

   (e) What was the difference in temperature between Athens and Montreal?

   (f) The average temperature in Samoa is 30°C. How much warmer was this than:

   (i) Helsinki (ii) London (iii) Moscow?



Unit 11: GEOMETRY – PART 2

In this unit you will be:

11.1 Using Bearings to Describe Direction and Plot Courses

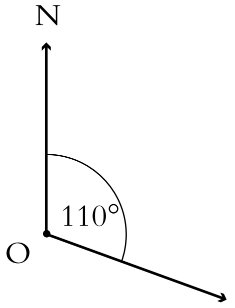
11.2 Using Angle Properties of Polygons to Solve Problems

- ◧ Revising Angles.
- ◧ Using Angle Properties of Polygons.

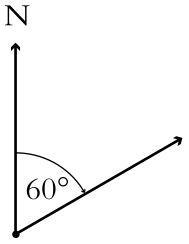
Section 11.1 Using Bearings To Describe Direction And Plot Courses

Bearings are a measure of direction, with north taken as a reference. If you are travelling north, your bearing is 000°.

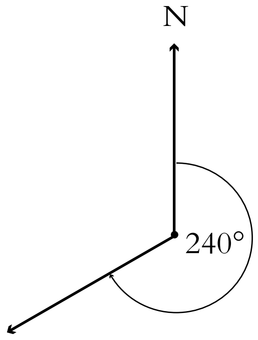
If you walk from O in the direction shown in the diagram, you are walking on a bearing of 110°.



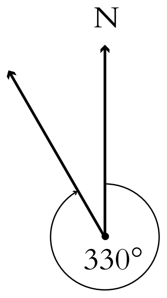
Bearings are always measured clockwise from north, and are given as three figures, for example:



Bearing 060°



Bearing 240°



Bearing 330°

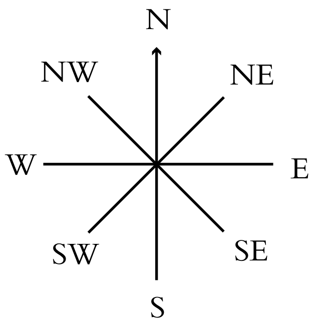
Example 1

On what bearing is a ship sailing if it is heading:

(a) E? (b) S?

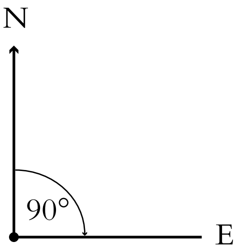
(c) W? (d) SE?

(e) NW?



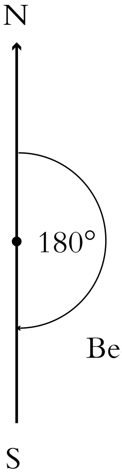
Solution

(a)



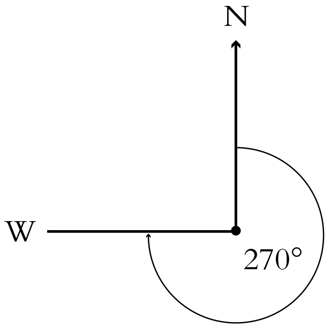
Bearing is 090°.

(b)



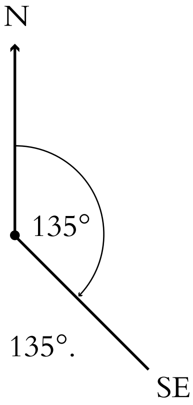
Bearing is 180°.

(c)



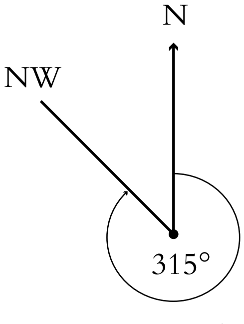
Bearing is 270°.

(d)



Bearing is 135°.

(e)



Bearing is 315°.

Example 2

A ships sails from A to B on a bearing of 060°. On what bearing must it sail if it is to return from B to A?

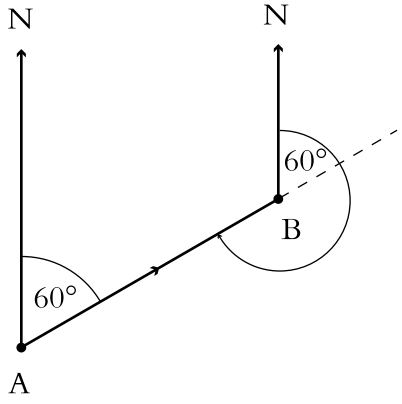
Solution

The diagram shows the journey from A to B.

Extending the line of the journey allows an angle of 60° to be marked at B.

Bearing of A from B $= 60° + 180°$

$= 240°$



Skill Exercises: Using Bearings

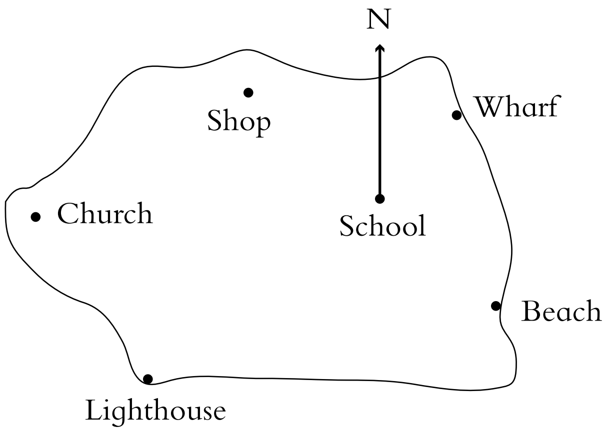
1. What angle do you turn through if you turn clockwise from:

   (a) N to S?
   (b) E to W?
   (c) N to NE?
   (d) N to SW?
   (e) W to NW?

2. Copy and complete the table:

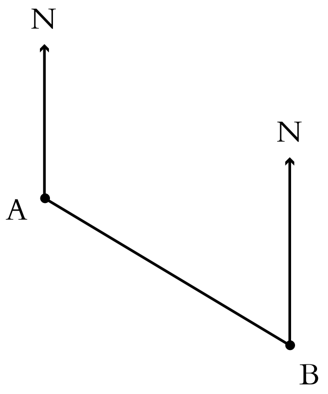
Direction	Bearing
N	
NE	
W	
SW	

3. The map of an island is shown below:



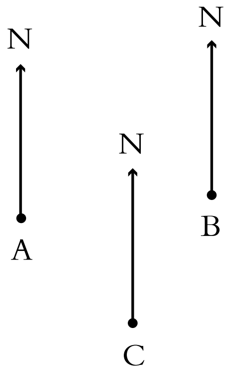
What is the bearing from the school, to each place shown on the map?

4. The diagram shows the positions of two ships, A and B.
   (a) What is the bearing of Ship A from Ship B?
   (b) What is the bearing of Ship B from Ship A?



5. The diagram shows 3 places, A, B and C.

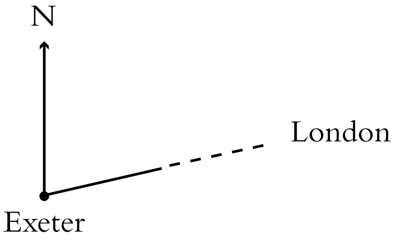
   Find the bearing of:
   (a) A from C.
   (b) B from A.
   (c) C from B.
   (d) B from C.



6. An aeroplane flies from Maota to Faleolo on a bearing of 044°. On what bearing should the pilot fly, to return to Maota from Faleolo?

7. On four separate occasions, a plane leaves Exeter airport to fly to a different destination. The bearings of these destinations from Exeter airport are given below.

Destination	Bearing
London	077°
Glasgow	356°
Leeds	036°
Guernsey	162°



   Copy and complete the diagram to show the direction in which the plane flies to each destination.

8. A ship sails NW from a port to take supplies to an oil rig. On what bearing must it sail to return from the oil rig to the port?

9. If A is north of B, and C is southeast of B on a bearing of 160° from A, find the bearing of:
   (a) A from B.
   (b) A from C.
   (c) C from B.
   (d) B from C.

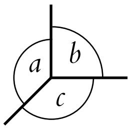
10. If A is on a bearing of 300° from O, and O is NE of B, and the bearing of B from A is 210°, find the bearing of:
    (a) A from B.
    (b) O from A.
    (c) O from B.

Section 11.2 Using Angle Properties Of Polygons To Solve Problems

Revising Angles

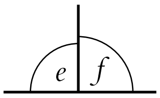
In this section we revise some basic work with angles, and begin by using the three rules listed below:

The angles at a point add up to 360°, e.g.



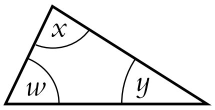
$$a + b + c = 360°$$

The angles on a straight line add up to 180°, e.g.



$$e + f = 180°$$

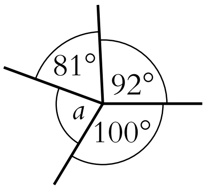
The angles in a triangle add up to 180°, e.g.



$$w + x + y = 180°$$

Example 1

Determine the size of angle a in the diagram shown:



Solution

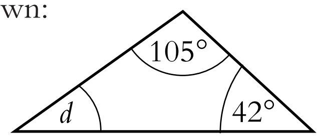
$81° + 92° + 100° + a = 360°$ (angle sum at a point)

$a + 273° = 360°$

$a = 87°$

Example 2

Determine the size of angle d in the diagram shown:



Solution

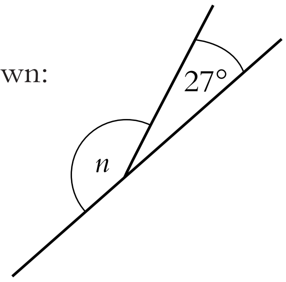
$105° + 42° + d = 180°$ (angle sum in a triangle)

$174° + d = 180°$

$d = 33°$

Example 3

Determine the size of angle n is the diagram shown:



Solution

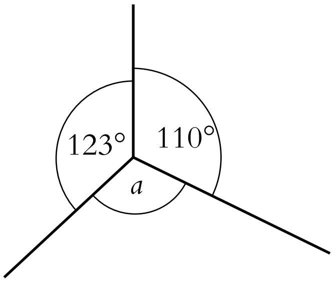
$n + 27° = 180°$ (angle sum on a line)

$n = 153°$

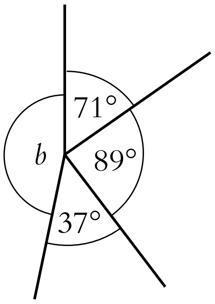
Skill Exercises: Revising Angles

1. Calculate the sizes of the angles marked by letters in the following diagrams:

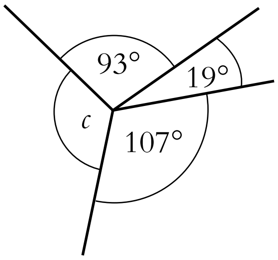
(a)



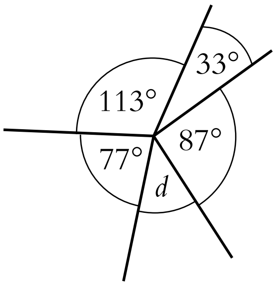
(b)



(c)

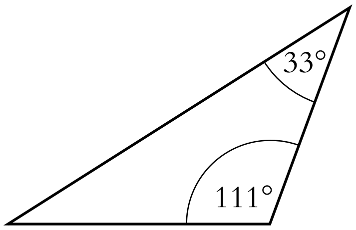


(d)

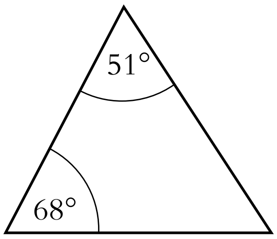


2. Calculate the sizes of the unknown angles in the following triangles:

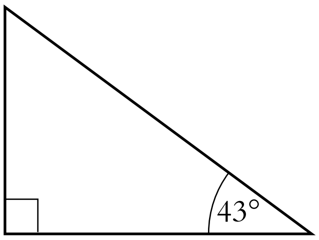
(a)



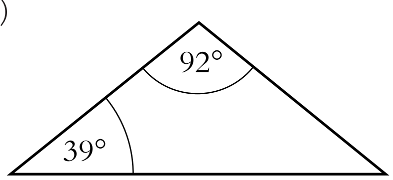
(b)



(c)

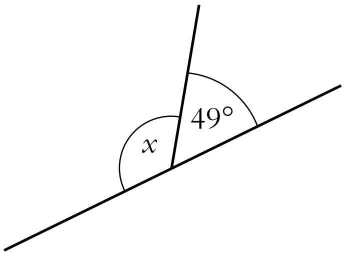


(d)

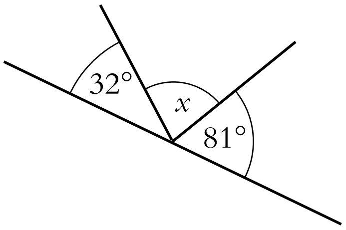


3. Calculate the sizes of the angles marked by the letter *x* in the following diagrams:

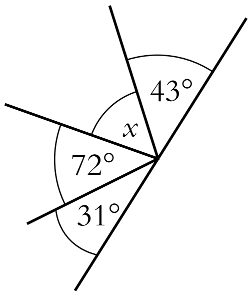
(a)



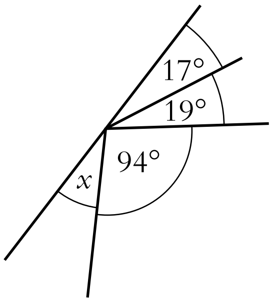
(b)



(c)

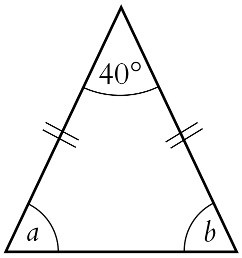


(d)

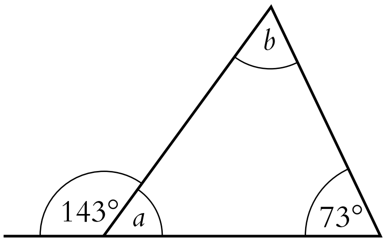


4. The diagram shows an isosceles triangle.

What are the sizes of the two angles marked *a* and *b*?

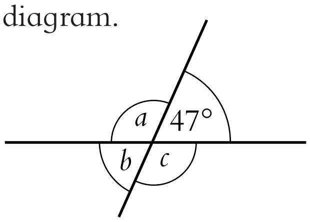


5. Calculate the sizes of the angles marked *a* and *b* in the diagram.

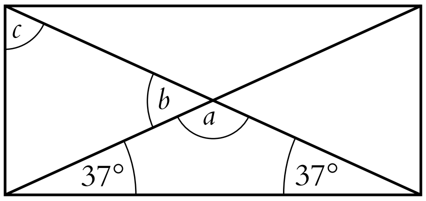


6. The diagram below shows two intersecting straight lines. Calculate the sizes of the angels marked *a*, *b* and *c* in the diagram.

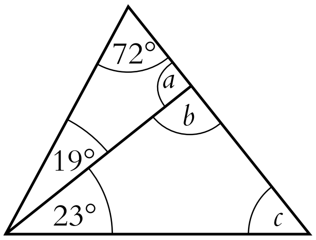
   What do you notice about angles *a* and *c*?



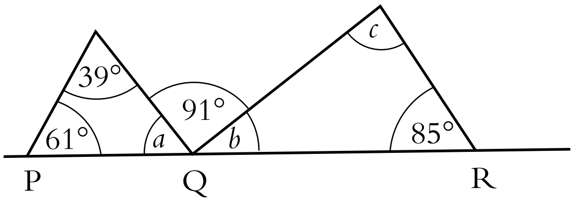
7. The diagram opposite shows a rectangle and its diagonals. Calculate the sizes of the angles marked *a*, *b* and *c*.



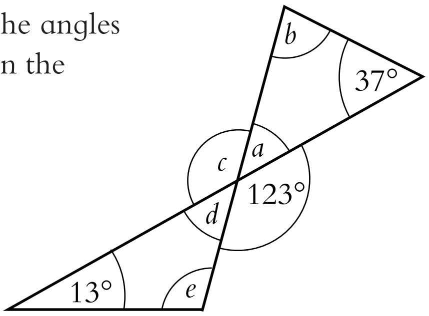
8. Determine the sizes of the angles marked *a*, *b* and *c* in the diagram shown.



9. PQR is a straight line. Determine the sizes of the angles marked *a*, *b* and *c* in the triangles shown.



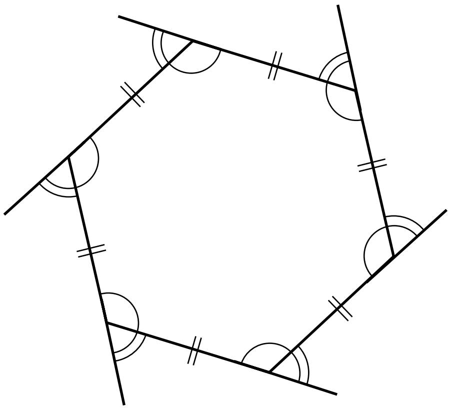
10. Calculate the sizes of the angles marked *a*, *b*, *c*, *d* and *e* in the triangles shown.



Using Angle Properties of Polygons

In this section we calculate the size of the interior and exterior angles for different regular polygons.

The following diagram shows a regular hexagon:



The angles marked ∠ are the interior angles of the hexagon.

The angles marked ∠ are the exterior angles of the hexagon.

In a regular polygon the sides are all the same length and the interior angles are all the same size.

Note that for any polygon: interior angle + exterior angle = 180°.

Since the interior angles of a regular polygon are all the same size, it follows that the exterior angles are also equal to one another.

One complete turn of the hexagon above will rotate any one exterior angle to each of the others in turn, which illustrates the following result:

The exterior angles of any polygon add up to 360°.

Example 1

Calculate the sizes of the interior and the exterior angles of a regular hexagon. Hence determine the sum of the interior angles.

Solution

The exterior angles of a regular hexagon are all equal, as shown in the previous diagram.

$$\begin{aligned}
\text{Therefore the exterior angle of a regular hexagon} &= \frac{360^\circ}{6} \\
&= 60^\circ \\
\text{Therefore the interior angle of a regular hexagon} &= 180^\circ - 60^\circ \\
&= 120^\circ \\
\text{The sum of the interior angles} &= 6 \times 120^\circ \\
&= 720^\circ
\end{aligned}$$

Example 2

The exterior angle of a regular polygon is 40°.

Calculate:

(a) The size of the interior angle.

(b) The number of sides of the polygon.

Solution

(a) Interior angle + exterior angle $= 180°$

Interior angle $= 180° - 40°$

$= 140°$

(b) the number of sides can be determined by dividing 360° by the size of the exterior angels, giving

$$\frac{360}{40} = 9$$

therefore the polygon has 9 sides.

In a regular polygon:

$$\text{Exterior angle} = \frac{360°}{\text{the number of sides}}$$

$$\text{Number of sides} = \frac{360°}{\text{exterior angle}}$$

Skill Exercises: Using Angle Properties of Polygons

1. Calculate the size of the exterior angels of a regular polygon which has interior angels of:
   - (a) 150°.
   - (b) 175°.
   - (c) 162°.
   - (d) 174°.

2. Calculate the sizes of the exterior and interior angles of:
   - (a) a regular octagon.
   - (b) a regular decagon.

3. (a) Calculate the size of the interior angles of a regular 12-sided polygon.

   (b) What is the sum of the interior angles of a regular 12-sided polygon?

4. (a) What is the size of the interior angles of a regular 20-sided polygon?

   (b) What is the sum of the interior angles of a regular 20-sided polygon?

5. Calculate the size of the exterior angle of a regular pentagon.

6. The size of the exterior angle of a regular polygon is 12°. How many sides does this polygon have?

7. Calculate the number of sides of a regular polygon with interior angles of:

   (a) (i) 150° (ii) 175° (iii) 162° (iv) 174°

   (b) Show why it is impossible for a regular polygon to have an interior angle of 123°.

8. (a) Complete the following table for regular polygons. Note that many of the missing values can be found in the examples and earlier exercises for this unit.

Number of Sides	Exterior Angles	Interior Angles	Sum of Interior Angles
4	90°		
5			
6			
7			
8			
9			
10			
12			

   (b) Describe an alternative way to calculate the sum of the interior angles of a regular polygon.

   (c) Draw and measure the angles in some irregular polygons. Which of the results in the table are also true for irregular polygons?

9. The exterior angle of a regular polygon is 4°.

   (a) How many sides does the polygon have?

   (b) What is the sum of the interior angles of the polygon?

10. A regular polygon has n sides.

   (a) Explain why the exterior angles of the polygon are of size $\frac{360^\circ}{n}$.

   (b) Explain why the interior angles of the polygon are $180^\circ - \frac{360^\circ}{n}$.

   (c) Write an expression for the sum of the interior angles.

Unit 8: ANSWERS – MEASUREMENT – PART 2

Section 8.1 Calculating And Using Rates

(Pg. 7) Skill Exercises: Average Speed

1. 40 km/h

2. 22.5 km/h

3. 4 m/hr

4. (a) 43 km/h (b) 56 km/h (c) 37.5 km/h

   (d) 48 km/h (e) 35 km/h

5. 20 km/h

6. 8 km/h

7.

	Julie	Debbie
(a)	25 wpm	20 wpm
(b)	1500 wphr	1200 wphr

8. (a) Salote 100 km/h (b) Ema 50 km/h

   (c) Ene 66.67 km/h

9. (a) 100 km/h (b) 132 km/h

   (c) 65 km/h

10. 63 km/h

(Pg. 10) Skill Exercises: Other Rates

1. 12 hr

2. (a) $1.98 (b) $2.76 (c) $4.71 (d) $9.39

3. (a) 93 sene
   (b) $8.56

4.

	Hours	Pay	Pay/Hour
Dee	8	28.64	3.58
Nadina	12	43.44	3.62
Lisa	42	302.40	7.20
Mary	38	136.80	3.60
Clare	35	134.40	3.84

   (a) Most, $7.20, Lisa
   (b) Least, $3.58, Dee

5. 0.2 litres/m^2

6. (a) $353 (U.S dollar)
   (b) $2182 (Tala)

Section 8.2 Calculating And Comparing Unit Costs

(Pg. 12) Skill Exercises: Calculating and Comparing Unit Costs

1. (a) Brand *x* $6 per litre
   Brand *y* $7 per litre
   (b) Brand *x* gives better value

2. Shop A 38 sene per kilogram
   Shop B 39.5 sene per kilogram
   Therefore Shop A gives better value

3. 50 ml — 2.1 sene per ml
   125 ml — 1.4 sene per ml
   175 ml — 1.2 sene per ml
   The best value is the 175 ml size

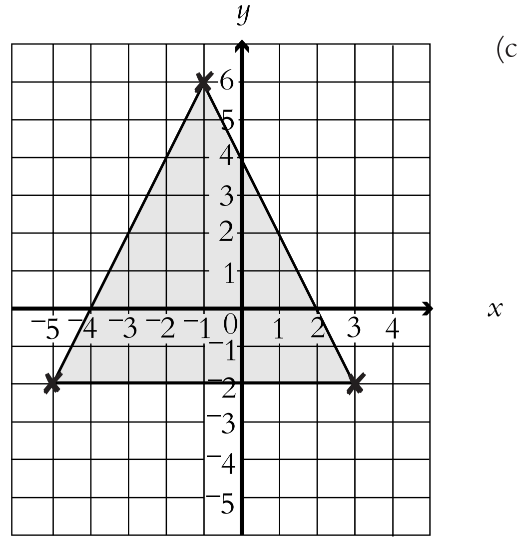
Unit 9: ANSWERS – ALGEBRA – PART 2

Section 9.1 Drawing Linear Graphs

(Pg. 15) Skill Exercises: Plotting Co-ordinates

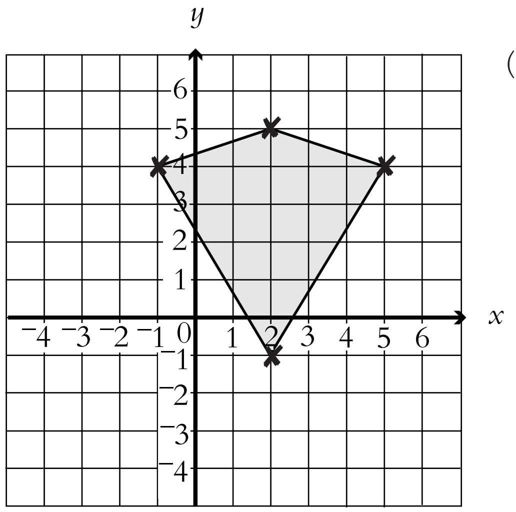
1. A(4, 8) B(5, $^-3$) C(3, $^-7$) D($^-5$, $^-2$) E($^-2$, 5) F(7, 4)
G(3, $^-2$) H($^-3$, $^-6$) I($^-6$, $^-5$) J($^-5$, 2) K($^-5$, 8)

2. (a), (b)



(c) isosceles

3. (a), (b)

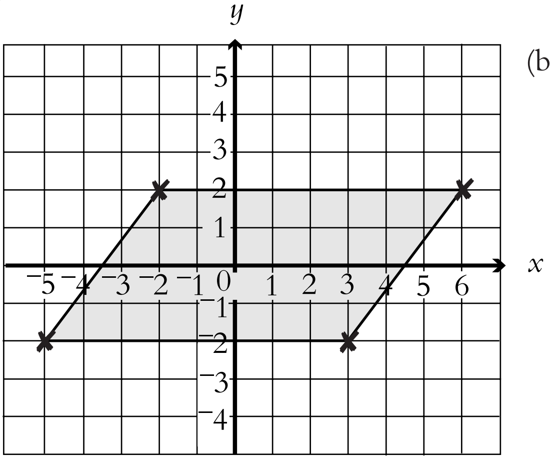


(c) kite

4. $(^-1, ^-3)$

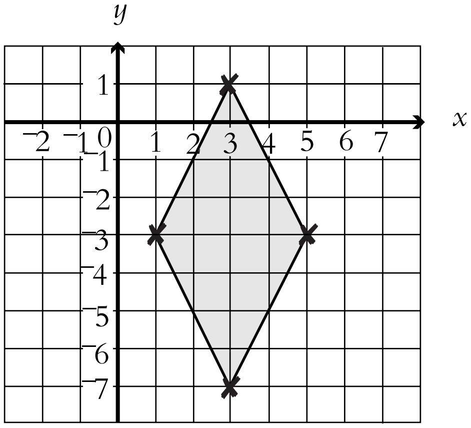
5. $(^-1, ^-5)$

6. (a)



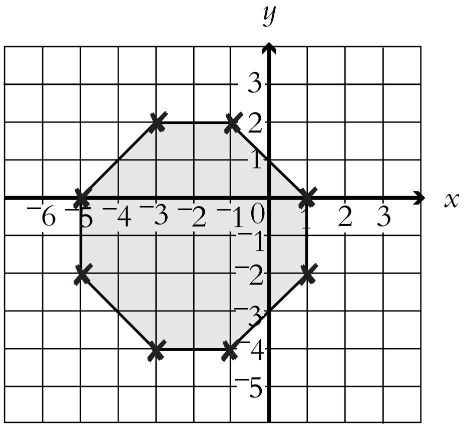
(b) parallelogram

7. (a)



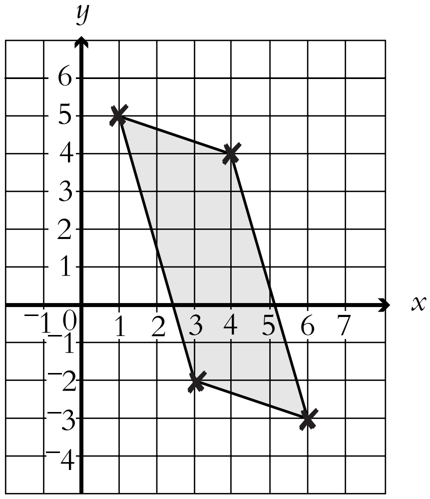
(b) rhombus

8. (a)



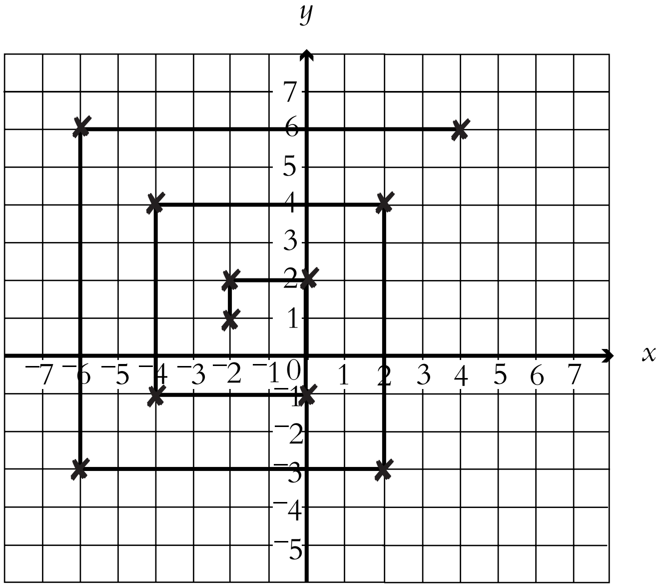
(b) octagon

9. (a)



(b) (3, −2)

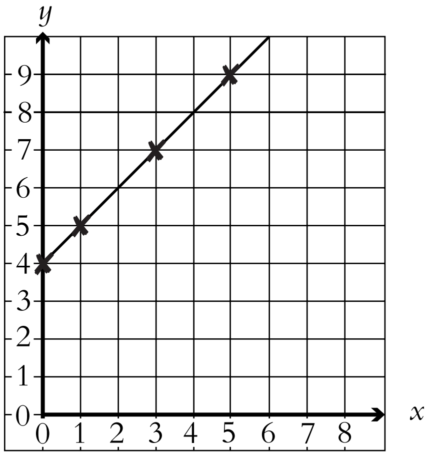
10. (a)



(b) (−6, −3), (−6, 6), (4, 6)

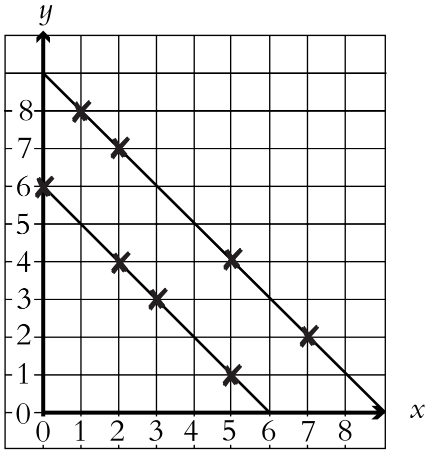
(Pg. 18) Skill Exercises: Plotting Points on a Straight Line

1. (a), (b)



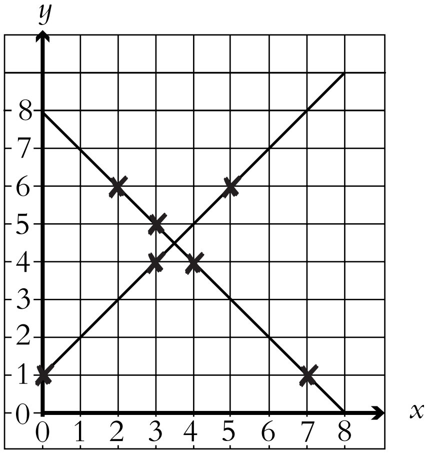
(c) (2, 6), (4, 8), (6, 10)

2. (a), (b)



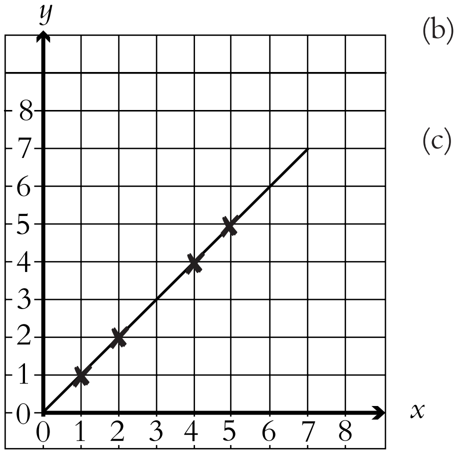
(c) These two lines are parallel.

3. (a), (b)



(c) These two lines are perpendicular.

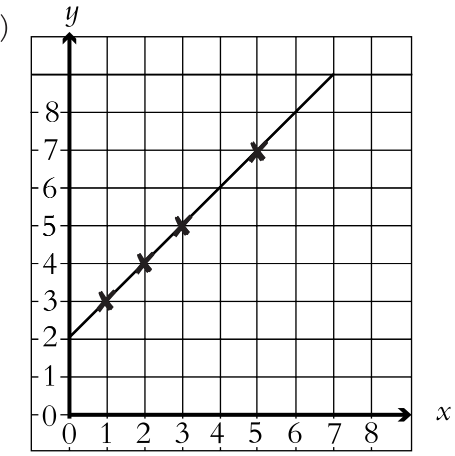
4. (a)



(b) For example (0, 0), (3, 3), (6, 6)

(c) The y co-ordinate is the same as (equals) the x co-ordinate i.e $y = x$

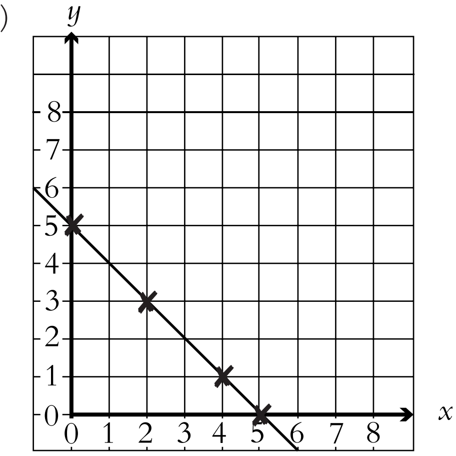
5. (a)



(b) For example (0, 2), (4, 6), (6, 8)

(c) The *y* co-ordinates is always 2 more than the *x* co-ordinates, i.e. $y = x + 2$

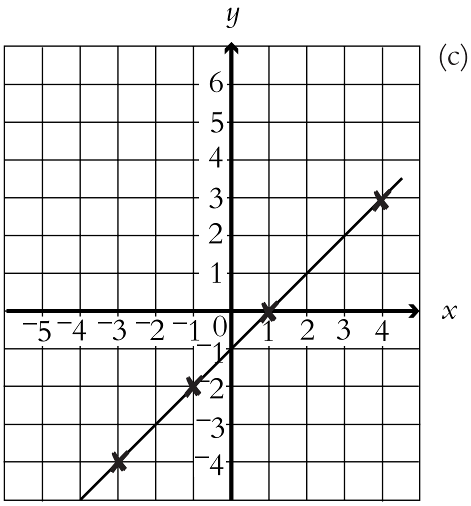
6. (a)



(b) For example, (1, 4), (3, 2)

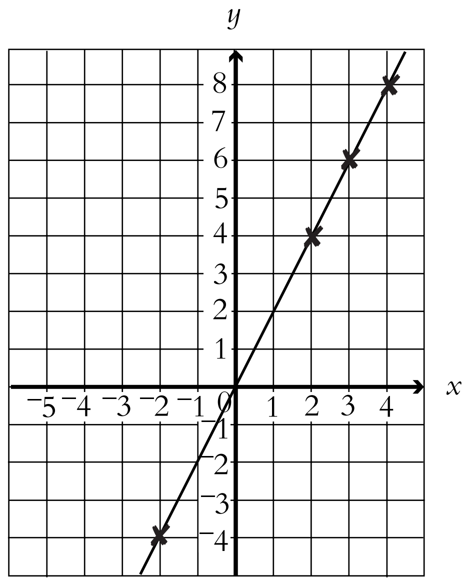
(c) 5

7. (a), (b)



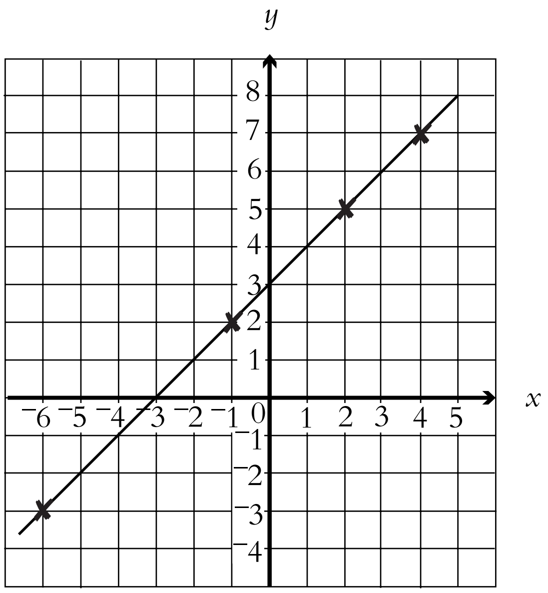
(c) The *y* co-ordinates is always one less than the *x* co-ordinate, i.e. $y = x - 1$

8. (a)



(b) The y co-ordinate is always twice the x co-ordinate, i.e. $y = 2x$

9. (a)

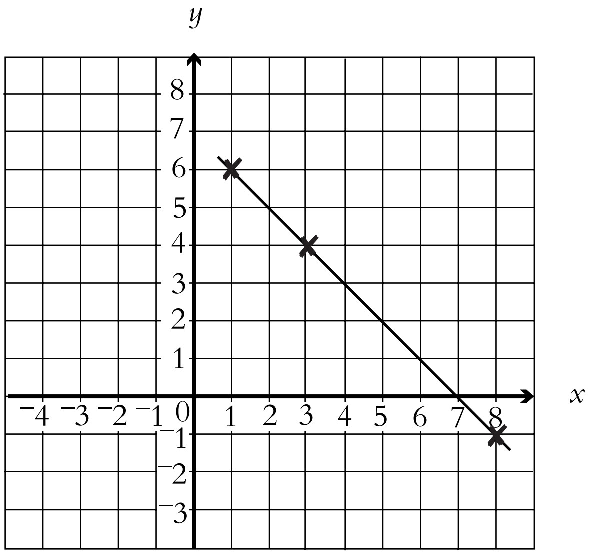


(b) $(^-7, ^-4)$, $(^-4, ^-1)$, $(3, 6)$, $(1, 4)$, $(100, 103)$

(c) The y co-ordinate is always 3 more than the x co-ordinate i.e. $y = x + 3$

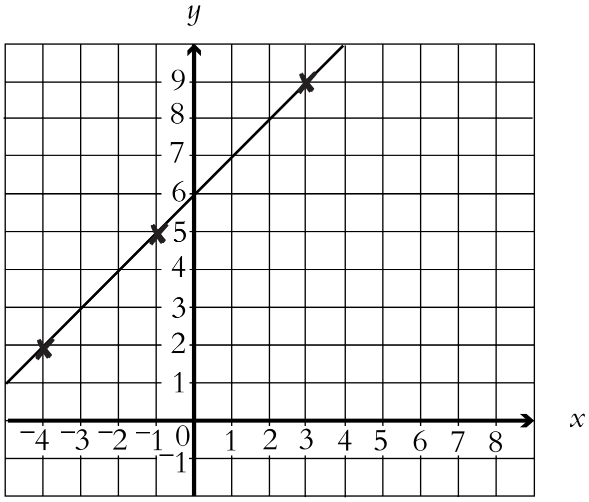
(d) No. 27 is only 2 more than 25.

10. (a)



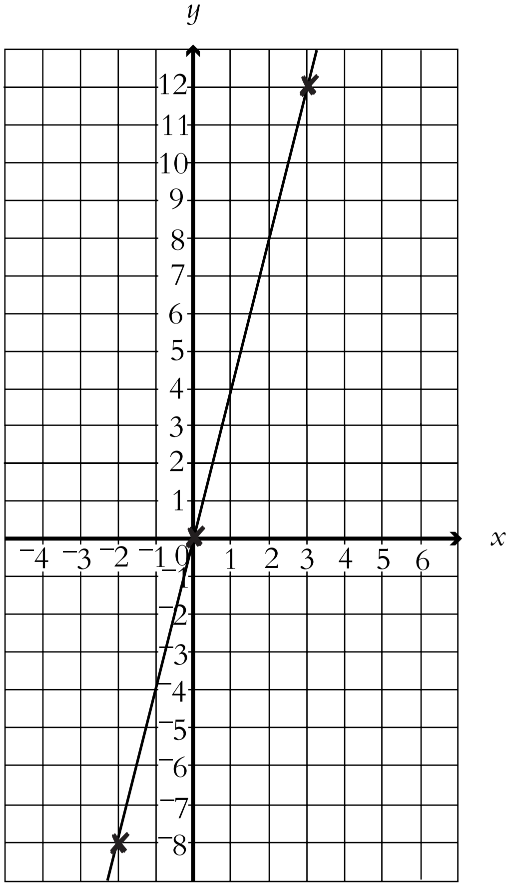
$x + y = 7$

(b)



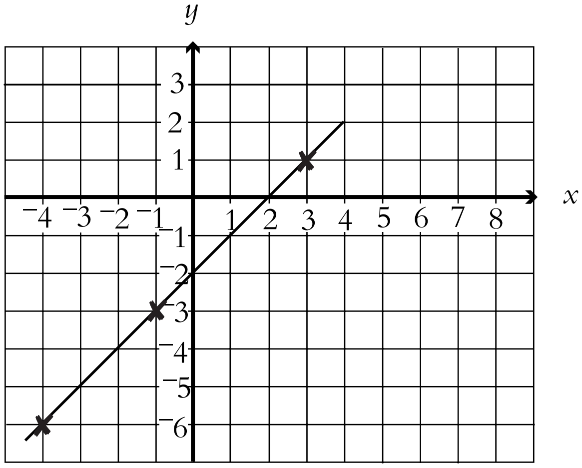
$y = x + 6$

(c)



$y = 4x$

(d)



$y = x – 2$

(Pg. 23) Skill Exercises: Drawing Graphs given their Equations

1. Positive: A, C, F   Negative: B, D E

2. (a) 4   (b) 1   (c) 5   (d) 2

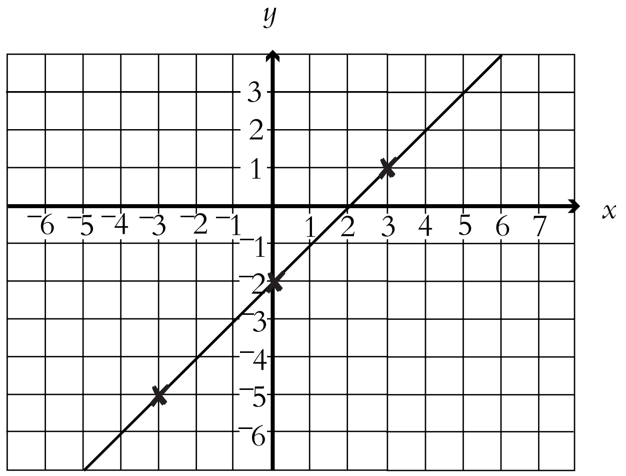
(e) $1\frac{1}{2}$ (f) 1   (g) $2\frac{1}{2}$ (h) 0

3. (a) $^{-}1$ (b) $^{-}4$ (c) $^{-}3$ (d) $^{-}2$

4. (a)

x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}5$	$^{-}4$	$^{-}3$	$^{-}2$	$^{-}1$	0	1

(b)

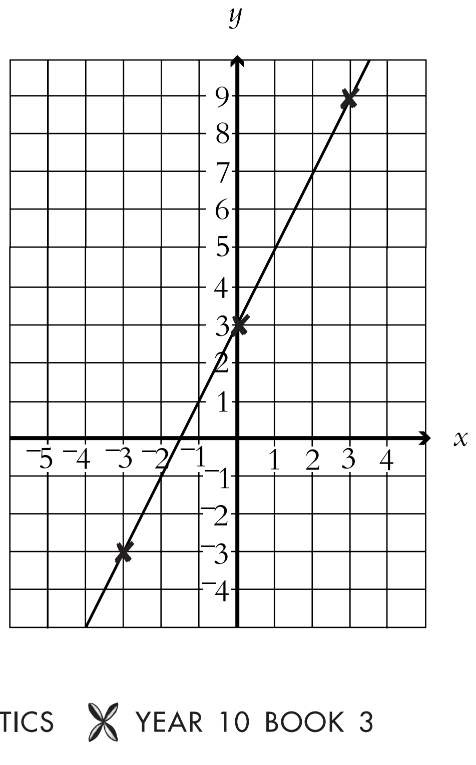


$y = x - 2$

5. (a)

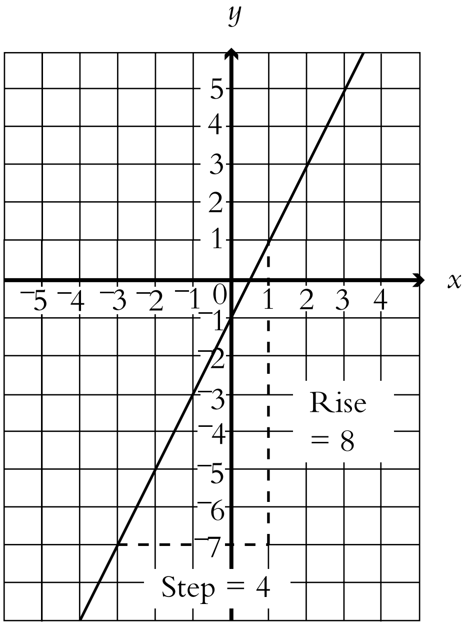
x	$^{-}3$	$^{-}2$	$^{-}1$	0	1	2	3
y	$^{-}3$	$^{-}1$	1	3	5	7	9

(b)



$y = 2x + 3$

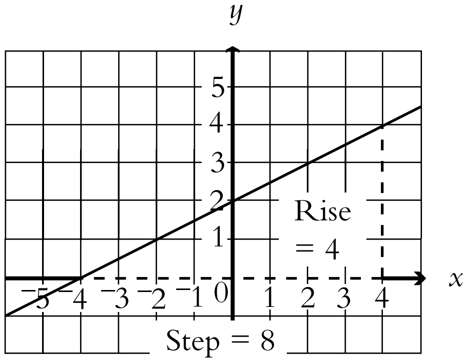
6. (a)



(b) Gradient $= \frac{8}{4}$

$= 2$

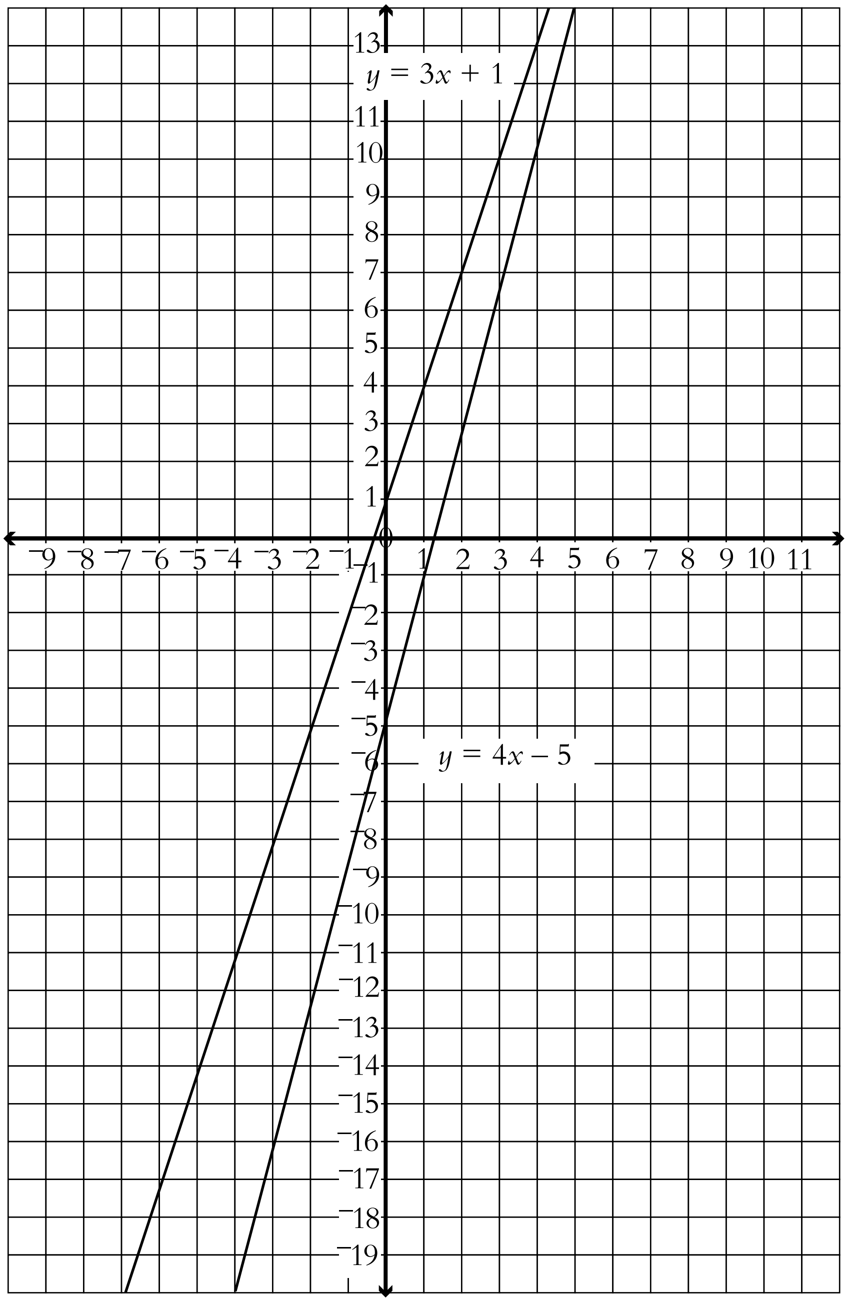
7. (a)



(b) Gradient $= \frac{4}{8}$

$= \frac{1}{2}$

8. (a)

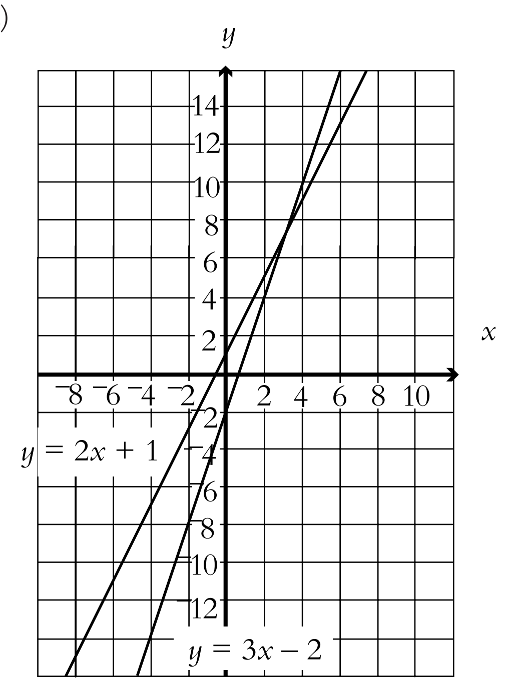


(b) $y = 3x + 1$
Gradient = 3

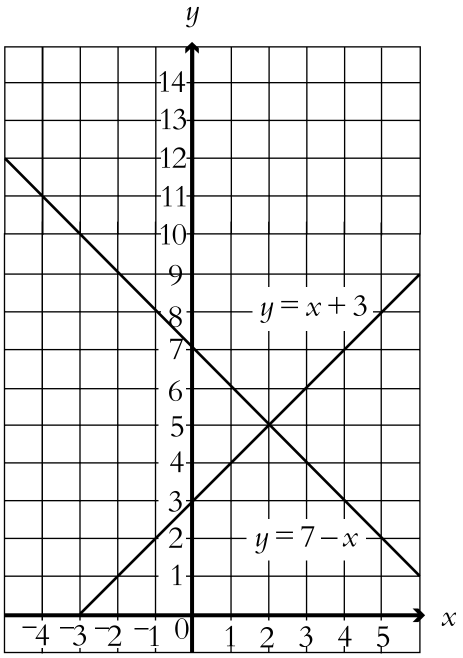
$y = 4x - 5$
Gradient = 4

9. (a) 2 (b) 3 (c) 10 (d) 5

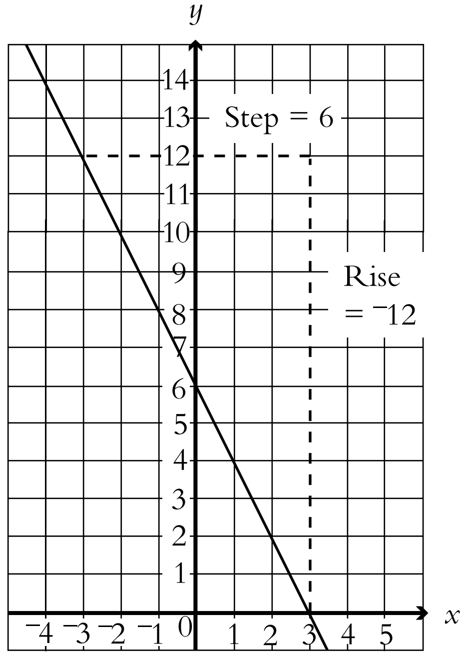
10. (a) (b) (3, 7)



11. Co-ordinates of intersection = (2, 5)



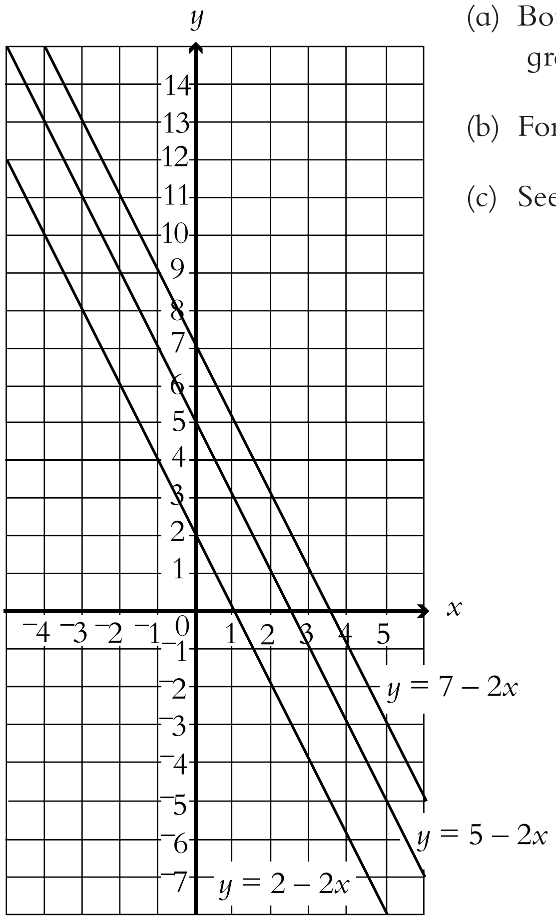
12. (a)



(b) Gradient $= \frac{^-12}{6}$

$= -2$

13.



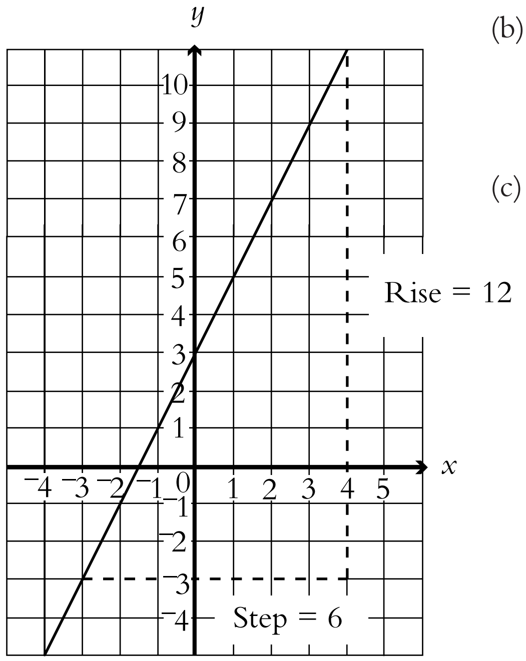
(a) Both lines have gradient = $^-2$

(b) For example $y = 7 - 2x$

(c) See diagram.

(Pg. 27) Skill Exercises: Finding the Equation of a Straight Line

1. (a)

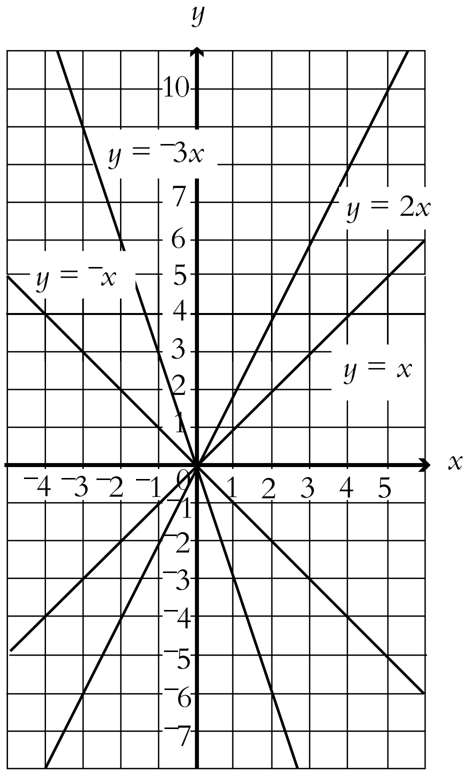


(b) Gradient $= \frac{12}{6}$

$= 2$

(c) Intercept $= 3$

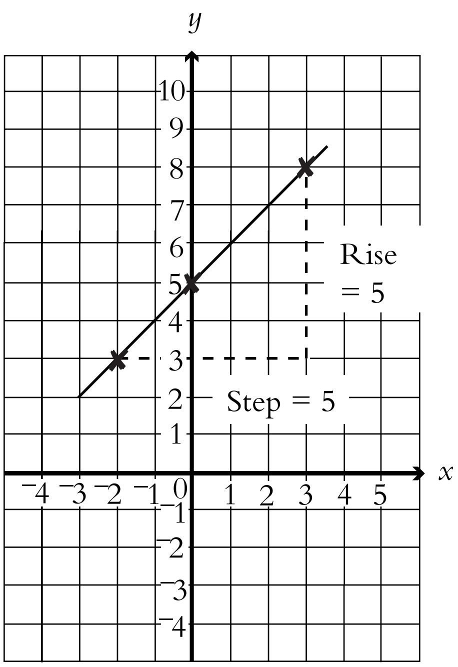
2. (a)



(b) Gradients $= 1, {}^{-}1,$
$2, {}^{-}3$

(c) $y = 0$

3. (a)



(b) Gradient $= \frac{\text{Rise}}{\text{Step}}$

$= 1$

(c) y intercept = 5

(d) $y = x + 5$

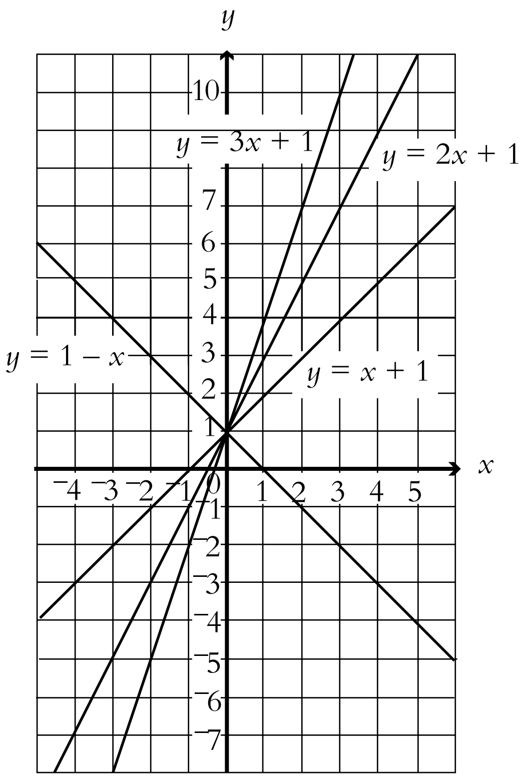
4.

	m	c	$y = mx + c$
(a)	1	⁻3	$y = x - 3$
(b)	2	2	$y = 2x + 2$
(c)	⁻1	4	$y = {}^{-}x + 4$ or $y = 4 - x$
(d)	⁻3	2	$y = {}^{-}3x + 2$ or $y = 2 - 3x$

5.

Equation	Gradient	Intercept
$y = 2x + 7$	2	7
$y = 8x - 2$	8	⁻2
$y = 8 - 3\text{x}$	⁻3	8
$y = 7x - 5$	7	⁻5
$y = {}^{-}3x + 2$	⁻3	2
$y = {}^{-}5x - 2$	⁻5	⁻2

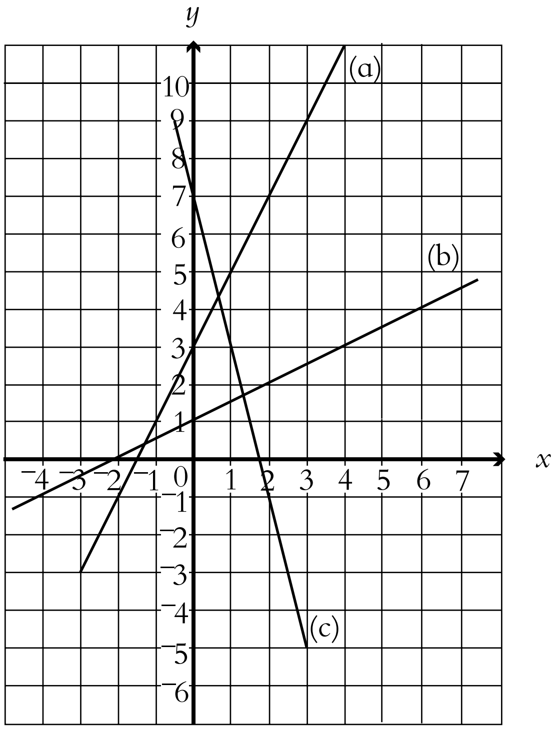
6. (a)



(b) All lines pass through $y = 1$ since $c = 1$ in each equation (same intercept).

7. (a) Gradient $= \frac{15}{5} = 3$ (b) $y = 0$ (c) $y = 3x$

8.



Section 9.2 Expanding And Factorising Expressions

(Pg. 30) Skill Exercises: Expanding Expressions with Two Brackets

1. (a)

×	x	5
x	x^2	$5x$
4	$4x$	20

$(x + 4)(x + 5)$
$= x^2 + 9x + 20$

×	x	$^-7$
x	x^2	^-7x
4	$4x$	$^-28$

$(x + 4)(x - 7)$
$= x^2 - 3x - 28$

×	x	4
x	x^2	$4x$
$^-1$	^-x	$^-4$

$(x - 1)(x + 4)$
$= x^2 + 3x - 4$

×	x	$^-5$
x	x^2	^-5x
$^-2$	^-2x	10

$(x - 2)(x - 5)$
$= x^2 - 7x + 10$

2. (a) $x^2 + 7x + 12$ (b) $x^2 + 3x - 10$ (c) $x^2 - 6x + 5$
(d) $x^2 + 4x - 21$ (e) $x^2 - x - 6$ (f) $x^2 + 3x - 4$

3. (a) $x^2 - 1$ (b) $x^2 - 4$ (c) $x^2 - 25$
(d) $x^2 - 49$ (e) no x terms

4. $(x + 5)^2 = (x + 5)(x + 5) = x^2 + 10x + 25$;

5. (a) $x^2 + 2x + 1$ (b) $x^2 - 2x + 1$ (c) $x^2 + 6x + 9$
(d) $x^2 - 10x + 25$

6. (a)

×	x	6
$2x$	$2x^2$	$12x$
1	x	6

(b) $2x^2 + 13x + 6$

7. (a) $4x^2 + 10x + 4$ (b) $12x^2 + 7x + 1$ (c) $6x^2 + x - 2$
(d) $20x^2 - x - 1$ (e) $4x^2 + 4x + 1$ (f) $16x^2 - 24x + 9$

8. (a) $13x$ (b) $12x$ (c) ^-7x
(d) $^-1$ (e) $2x^2$ (f) ^-14x

9. $(x + 4)(x - 5) = x^2 - x - 20$; no x terms in original statement

10. (a) 2 (b) 6 (c) 3, 9
(d) 7, 35 (e) 2, 2 (f) 4, 2 or 2, 4

(Pg. 33) Skill Exercises: Factorising Expressions

1. (a) $2(2x - 1)$ (b) $6(x - 2)$ (c) $5(x - 4)$
(d) $4(x + 8)$ (e) $2(3x - 4)$ (f) $4(2 - 3x)$
(g) $7(3x - 2)$ (h) $5(3x + 4)$ (i) $10(3 - x)$

2. (a) $x(x + 4)$ (b) $x(x - 3)$ (c) $x(4 - x)$
(d) $2x(3x + 4)$ (e) $3x(3x +5)$ (f) $7x(x - 3)$
(g) $7x(4 - 5x)$ (h) $2x(3x - 7)$ (i) $x(5x - 3)$

3. (a) $x^2(x + 1)$ (b) $x^2(2 - x)$ (c) $2x^2(2x - 1)$
(d) $4x^2(2x + 1)$ (e) $4x^2(4 - 9x)$ (f) $2x^2(2x + 11)$
(g) $2x^2(8 - 3x)$ (h) $7x^2(2x + 3)$ (i) $7x^2(4x - 7)$

4. (a) $x^2 - 25$ (b) $(x + 5)(x - 5)$
(c) (i) $(x + 7)(x - 7)$ (ii) $(x + 8)(x - 8)$ (iii) $(x + 10)(x - 10)$
(iv) $(x + a)(x - a)$ (v) $(x + 2b)(x - 2b)$

5. (a) $(x + 3)(x + 4)$ (b) $(x + 1)(x + 7)$ (c) $(x + 2)(x + 9)$
(d) $(x + 3)(x + 9)$ (e) $(x + 7)(x + 10)$ (f) $(x + 2)(x + 4)$
(g) $(x + 2)(x + 14)$ (h) $(x + 7)(x + 11)$ (i) $(x + 7)(x + 9)$

6. (a) $(x + 2)(x - 1)$ (b) $(x + 5)(x - 4)$ (c) $(x + 3)(x - 4)$
(d) $(x - 4)(x - 9)$ (e) $(x - 2)(x - 8)$ (f) $(x + 7)(x - 6)$
(g) $(x + 15)(x - 2)$ (h) $(x - 8)(x - 9)$ (i) $(x + 9)(x - 11)$

7. $a = x - 5$

8. $a = x + 5$

9. $h = x + 5$

10. $a = 10$

Unit 10: ANSWERS – STATISTICS AND PROBABILITY – PART 2

Section 10.1 Displaying Data

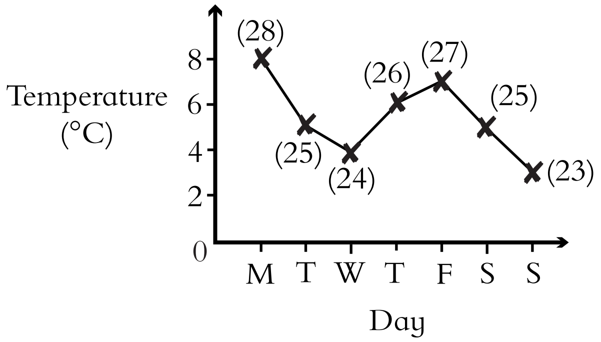
(Pg. 40) Skill Exercises: Displaying Data on a Line Graph

1. (a) 4 cm (b) April (c) December to February
   (d) July and August

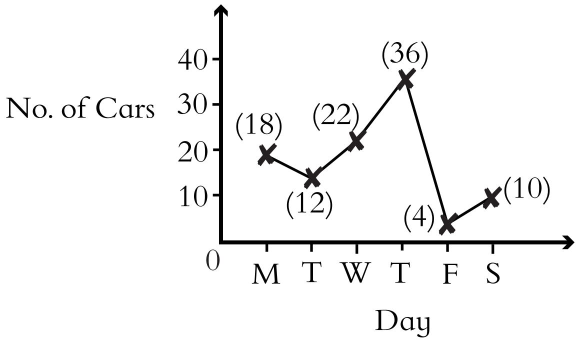
2. (a) 40°C (b) 80°C (c) 20 minutes after filling the cup
   (d) 25 minutes

3. (a) 8 cm (b) 22 cm (c) 84 cm (d) 3 weeks

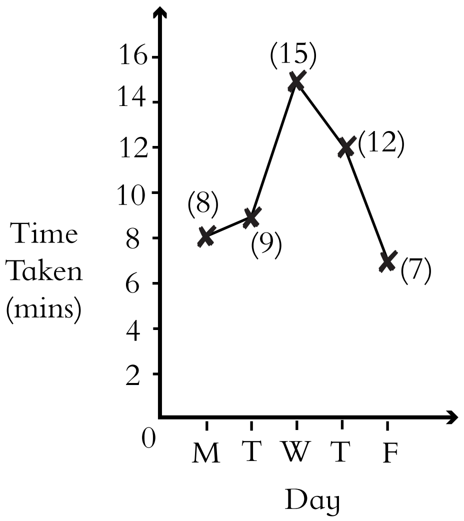
4.



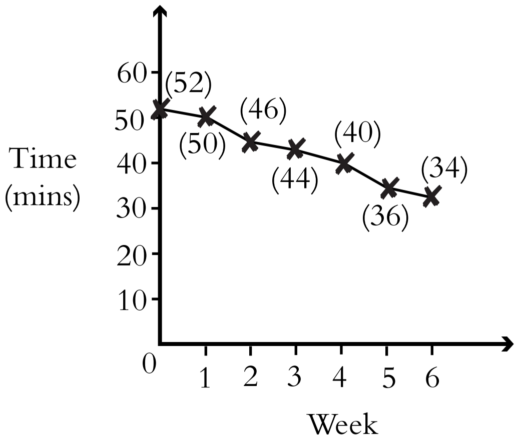
5.



6.



7.



(Pg. 43) Skill Exercises: Displaying Data on a Stem and Leaf Plot

In this stem and leaf plot we treat the numbers of tens as the stem and the numbers of units as the leaves.

1. In the following stem and leaf plot the data has not been put in order;

Stem	Leaf											
0	5	7										
1	7	7	9	9	2							
2	3	6	1	9								
3	1	1	2	6	7	7	8	9	8	2	6	5
4	6	2	1	0	8	2	1					

The leaves can now be ordered as shown to produce the final diagram:

Stem	Leaf
0	5 7
1	2 7 7 9 9
2	1 3 6 9
3	1 1 2 2 5 6 6 7 7 8 8 9
4	0 1 1 2 2 6 8

2.

Stem	Leaf
0	5 5 7 9 9
1	0 5 6 7 7 7 7 8 8 9 9
2	0 1 1 1 2 2 2 3
3	1

3.

Stem	Leaf
0	
1	5 7 8 8 8 9 9
2	0 1 1 1 1 2 3 3 3 4 4 5 6 7 7 8
3	1 2 3 4 5 5 6 7
4	0 3 5 5 8
5	1 7 8
6	1 2 3 5 5 9
7	1 3
8	2

Section 10.2 Analysing Data

(Pg. 47) Skill Exercises: Calculating Mean, Median, Mode and Range

1.

	Mean	Median	Mode	Range
(a)	5	4	3	6
(b)	11	11	12	5
(c)	7.5	7.5	none	5
(d)	5.5	5	5	7

2.

Score	Frequency	Score × Frequency
0	2	0
1	6	6
2	8	16
3	3	9
4	0	0
5	1	5
Totals	20	36

(b) Mean $= \frac{36}{20}$

$= 1.8$

3.

No. of Goals	Tally	Frequency	No. of Goals × Frequency
0	𝍸	5	0
1	𝍸 𝍸	10	10
2	𝍸 ||	7	14
3	||||	4	12
4	||	2	8
5	|	1	5
6	|	1	6
Totals	30	30	55

(b) Mean = 1.83 (2 d.p)

(c) Median = 1.5

(d) Mode = 1 goal

(e) Range = 6

4. Mean = \$1.77 sene   Median = \$1.77 sene   Mode = \$1.77 sene

5. (a) Mean = 2.1 children   (b) Median = 2 children

(c) The survey is of a class. Each family must have at least one child.

6. Mean = 72

7. (a) Mean = 5.24 Median = 5 Mode = 4

(b) Mode value

8. (a) Mean = 2.44 vacuum cleaners

Median = 2 vacuum cleaners

Mode = 4 vacuum cleaners

(b) Mode

9. (a)

	Class A	Class B
Mean	6.88	6.88
Median	7	7
Mode	6 and 8	10

(b) None

(c) Mode

10.

	Mean	Median	Mode
Paulo	72.65	72	72
Tavita	73.15	71	70

Paulo is the better player because his mean is 72.65 compared with Tavita's 73.15.

Tavita is the better player because his median score is 71 compared with Paulo's 72 and also he scores 70 (the mode) more often than Paulo who gets 72 most frequently.

Section 10.3 Reporting Statistical Findings

(Pg. 51) Skill Exercises: Statistical Findings

1. (a) Pita 175cm (b) Tama was second

(c) Sione and Simiona (d) 20 cm

2. (a) 4 points (b) Vaiala and Moataa

(c) Marist

3. (a) (i) Los Angeles (ii) Moscow

(b) 21°C

(c) New York

(d) London

(e) 18°C

(f) (i) 30°C (ii) 24°C (iii) 36°C

Unit 11: ANSWERS – GEOMETRY – PART 2

Section 11.1 Using Bearings To Describe Direction And Plot Courses

(Pg. 58) Skill Exercises: Using Bearings

1. (a) 180° (b) 180° (c) 45°
   (d) 225° (e) 45°

2.

Direction	Bearing
N	000°
NE	045°
W	270°
SW	225°

3.

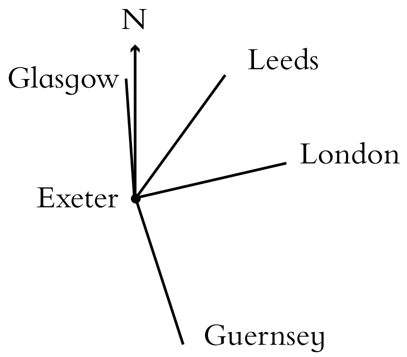
Location	Bearing
Wharf	043°
Beach	133°
Lighthouse	232°
Church	267°
Mine	309°

4. (a) 300° (b) 060°

5. (a) 308° (b) 083° (c) 212° (d) 032°

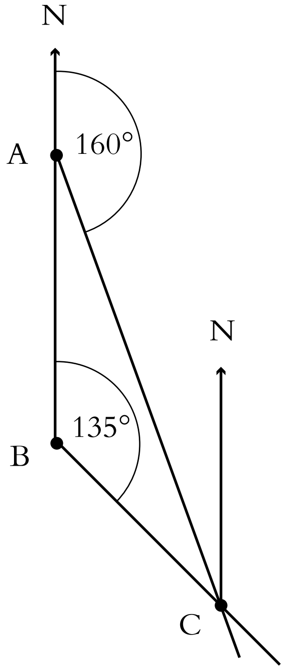
6. 044° + 180° = 224°

7.

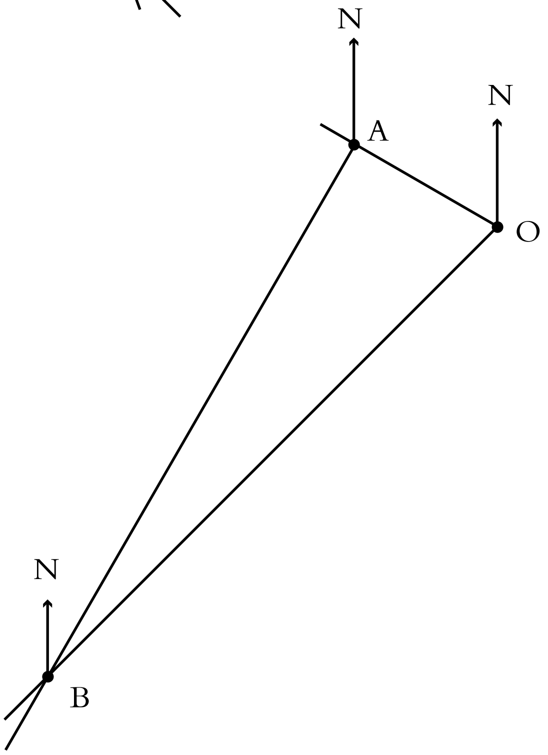


8. Sails 135° on return (SE).

9. (a) 000°
   (b) 340°
   (c) 135°
   (d) 315°



10. (a) 030°
    (b) 120°
    (c) 045°



Section 11.2 Using Angle Properties Of Polygons To Solve Problems

(Pg. 61) Skill Exercises: Revising Angles

1. (a) 127° (b) 163° (c) 141° (d) 50°

2. (a) 36° (b) 61° (c) 47° (d) 49°

3. (a) 131° (b) 67° (c) 34° (d) 50°

4. $a = b = 70^\circ$

5. $a = 37^\circ$, $b = 70^\circ$

6. $b = 47^\circ$, $a = c = 133^\circ$ (a and c are equal — vertically opposite)

7. $a = 106^\circ$, $b = 74^\circ$, $c = 53^\circ$

8. $a = 89^\circ$, $b = 91^\circ$, $c = 66^\circ$

9. $a = 80^\circ$, $b = 9^\circ$, $c = 86^\circ$

10. $a = 57^\circ$, $b = 86^\circ$, $c = 123^\circ$, $d = 57^\circ$, $e = 110^\circ$

(Pg. 65) Skill Exercises: Using Angle Properties of Polygons

1. (a) 30° (b) 5° (c) 18° (d) 6°

2. (a) Exterior = 45°, Interior = 135°

   (b) Exterior = 36°, Interior = 144°

3. (a) 150° (b) 1800°

4. (a) 162° (b) 3240°

5. 72°

6. 30 sides

7. (a) (i) 12 (ii) 72 (iii) 20 (iv) 60
   (b) Interior angle = 123°, Exterior angle = 57°

   The number of sides = $\frac{360}{57}$ which is not an exact integer. Therefore

   a regular polygon is not possible.

8. (a)

Number of Sides	Exterior Angles	Interior Angles	Sum of Interior Angles
4	90°	90°	360°
5	72°	108°	540°
6	60°	120°	720°
7	$51\frac{3}{7}°$	$128\frac{4}{7}°$	900°
8	45°	135°	1080°
9	40°	140°	1260°
10	36°	144°	1440°
12	30°	150°	1800°

(b) If n = no. of sides of regular polygon, sum = $(n - 2) \times 180°$.

(c) Sum of interior angles.

9. (a) 90 (b) $(90 - 2) \times 180 = 15\,840°$

10. (a) $\dfrac{360°}{\text{no. of sides}}$

(b) Interior angle $= 180° - \text{exterior angle}$

$$= 180° - \frac{360°}{n}$$

(c) sum $= 180°(n - 2)$