Book 1

Year 10

Mathematics

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Year 10 Book One

GOVERNMENT OF SĀMOA
MINISTRY OF EDUCATION, SPORTS AND CULTURE

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Section 1.1 Using Inequality Signs

What are inequality signs?

An inequality is a mathematical sentence that states that one quantity is greater than or less than another in value.

These signs are used:

$>$	is greater than.
e.g. $7 > 3$	seven is greater than three.
$<$	is less than.
e.g. $3 < 7$	three is less than seven.
$\geq$	greater than or equal to.
e.g. $a \geq 5$	a is greater than or equal to five.
$\leq$	less than or equal to.
e.g. $b \leq 7$	b is less than or equal to seven.

Skill Exercises: Inequality Signs

Put the correct sign, $<$, $>$ or $=$ into each sentence.

1. (a) $5 \ldots 8$ (b) $15 \ldots 10$ (c) $7 + 3 \ldots 4 + 6$
   (d) $3 + 4 \ldots 5 + 1$

2. (a) $^{-}7 \ldots {}^{-}2$ (b) $3 - 2 \ldots {}^{-}5$ (c) $3 - 5 \ldots {}^{-}4 - 6$
   (d) $0 \ldots {}^{-}3$

3. If x is an element of $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ list the numbers that will make these sentences true.
   (a) $x < 6$ (b) $x \geq 8$ (c) $x \leq 4$ (d) $x > 9$

Section 1.2 Applying The Laws Of Exponents

What is an Exponent?

$$2^3$$

Exponent. It can also be called an index or a power.

Base Number.

We say this as 'two to the power of three'. The exponent tells us how many times$^{(3)}$ to multiply the base(2) by itself.

$$2^3 = 2 \times 2 \times 2$$

Example 1

Calculate the value of:

(a) 5^2 (b) 2^5 (c) 3^3

Solution

(a) $5^2 = 5 \quad 5$
$\quad = 25$

(b) $2^5 = 2 \quad 2 \quad 2 \quad 2 \quad 2$
$\quad = 32$

(c) $3^3 = 3 \quad 3 \quad 3$
$\quad = 27$

Example 2

Copy each of the following statements and fill in the missing number or numbers:

(a) $2^{\square} = 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$

(b) $9 = 3^{\square}$

(c) $1000 = 10^{\square}$

(d) $5^3 = \square \quad \square \quad \square$

Solution

(a) $2^7 = 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2 \quad 2$

(b) $9 = 3 \quad 3 = 3^2$

(c) $1000 = 10 \quad 10 \quad 10 = 10^3$

(d) $5^3 = 5 \quad 5 \quad 5$

Example 3

(a) Calculate 2^5

(b) Calculate 2^3

(c) Calculate $2^5 \quad 2^3$

(d) Express your answer to (c) in index form

Solution

(a) $2^5 = 32$

(b) $2^3 = 8$

(c) $2^5 \quad 2^3 = 32 \quad 8$
$\quad = 4$

(d) $4 = 2 \quad 2$
$\quad = 2^2$

Skill Exercises: Exponents

1. Calculate:

(a) 2^3 (b) 10^2 (c) 3^2

(d) 10^3 (e) 9^2 (f) 3^3

(g) 2^4 (h) 3^4 (i) 7^2

2. Copy each of the following statements and fill in the missing numbers:

(a) $10 \quad 10 \quad 10 \quad 10 \quad 10 = 10^{\square}$ (b) $3 \quad 3 \quad 3 \quad 3 = 3^{\square}$

(c) $7 \quad 7 \quad 7 \quad 7 \quad 7 = 7^{\square}$ (d) $8 \quad 8 \quad 8 \quad 8 \quad 8 = 8^{\square}$

(e) $5 \quad 5 = 5^{\square}$ (f) $19 \quad 19 \quad 19 \quad 19 = 19^{\square}$

(g) $6 \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 \quad 6 = 6^{\square}$

(h) $11 \quad 11 \quad 11 \quad 11 \quad 11 \quad 11 = 11^{\square}$

3. Copy each of the following statements and fill in the missing numbers:

(a) $8 = 2^{\square}$ (b) $81 = 3^{\square}$

(c) $100 = 10^{\square}$ (d) $81 = 9^{\square}$

(e) $125 = 5^{\square}$ (f) $1\,000\,000 = 10^{\square}$

(g) $216 = 6^{\square}$ (h) $625 = 5^{\square}$

4. Is 10^2 bigger than 2^{10}?

5. Is 3^4 bigger than 4^3?

6. Is 5^2 bigger than 2^5?

7. Copy each of the following statements and fill in the missing numbers:

(a) $49 = \square^2$ (b) $64 = \square^3$

(c) $64 = \square^6$ (d) $64 = \square^2$

(e) $100\,000 = \square^5$ (f) $243 = \square^5$

8. Calculate:

(a) $2^2 + 2^3$ (b) $2^2 \quad 2^3$ (c) $3^2 + 2^2$

(d) $3^2 \quad 2^2$ (e) $2^3 \quad 10^3$ (f) $10^3 + 2^5$

9. Calculate:

(a) $(3 + 2)^4$ (b) $(3 - 2)^4$

(c) $(7 - 4)^3$ (d) $(7 + 4)^3$

10. Writing your answers in index form, calculate:

(a) $10^2 \quad 10^3$ (b) $2^3 \quad 2^7$

(c) $3^4 \quad 3^2$ (d) $2^5 \quad 2^2$

(e) $10^6 \quad 10^2$ (f) $5^4 \quad 5^2$

11. (a) Without using a calculator, write down the values of k and m.

$64 = 8^2 = 4^k = 2^m$

(b) Complete the following:

$2^{15} = 32\ 768$

$2^{14} = \boxed{}$

The Laws of Exponents

There are four rules that should be used when working with exponents:

When m and n are positive integers,

Rule 1: $a^m \quad a^n \quad = \quad a^{m+n}$

Rule 2: $a^m \quad a^n \quad = \quad a^{m-n}$ or $\dfrac{a^m}{a^n} = a^{m-n}$

Rule 3: $(a^m)^n \quad = \quad a^{m \quad n}$

Rule 4: $a^0 \quad = 1$

Example 1

Fill in the missing numbers in each of the following expressions:

(a) $2^4 \quad 2^6 = 2^{\square}$

(b) $3^7 \quad 3^9 = 3^{\square}$

(c) $3^6 \quad 3^2 = 3^{\square}$

(d) $(10^4)^3 = 10^{\square}$

(e) $4^0 = \square$

Solution

(a) $2^4 \quad 2^6 = 2^{4+6}$ (Rule 1)
$= 2^{10}$

(b) $3^7 \quad 3^9 = 3^{7+9}$ (Rule 1)
$= 3^{16}$

(c) $3^6 \quad 3^2 = 3^{6-2}$ (Rule 2)
$= 3^4$

(d) $(10^4)^3 = 10^{4 \quad 3}$ (Rule 3)
$= 10^{12}$

(e) $4^0 = 1$ (Rule 4)

Example 2

Simplify each of the following expressions so that it is in the form a^n, where n is a number:

(a) $a^6 \quad a^7$ (b) $\dfrac{a^4 \quad a^2}{a^3}$ (c) $(a^4)^3$

Solution

(a) $a^6 \quad a^7 = a^{6+7}$
$= a^{13}$

(b) $\dfrac{a^4 \quad a^2}{a^3} = \dfrac{a^{4+2}}{a^3}$
$= \dfrac{a^6}{a^3}$
$= a^{6-3}$
$= a^3$

(c) $(a^4)^3 = a^{4 \quad 3}$
$= a^{12}$

Skill Exercises: The Laws of Exponents

1. Copy each of the following statements and fill in the missing numbers:

(a) $2^3 \quad 2^7 = 2^{\square}$ (b) $3^6 \quad 3^5 = 3^{\square}$

(c) $3^7 \quad 3^4 = 3^{\square}$ (d) $8^3 \quad 8^4 = 8^{\square}$

(e) $(3^2)^5 = 3^{\square}$ (f) $(2^3)^6 = 2^{\square}$

(g) $\dfrac{3^6}{3^2} = 3^{\square}$ (h) $\dfrac{4^7}{4^2} = 4^{\square}$

2. Copy each of the following statements and fill in the missing numbers:

(a) $a^3 \quad a^2 = a^{\square}$ (b) $b^7 \quad b^2 = b^{\square}$

(c) $(b^2)^5 = b^{\square}$ (d) $b^6 \quad b^4 = b^{\square}$

(e) $(z^3)^9 = z^{\square}$ (f) $\dfrac{q^{16}}{q^7} = q^{\square}$

3. Show why $9^4 = 3^8$.

4. Calculate:

(a) $3^0 + 4^0$ (b) $6^0 \quad 7^0$

(c) $8^0 - 3^0$ (d) $6^0 + 2^0 - 4^0$

5. Copy each of the following statements and fill in the missing numbers:

(a) $3^6 \quad 3^{\square} = 3^{17}$ (b) $4^6 \quad 4^{\square} = 4^{11}$

(c) $\dfrac{a^6}{a^{\square}} = a^4$ (d) $(z^{\square})^6 = z^{18}$

(e) $(a^{19})^{\square} = a^{95}$ (f) $p^{16} \quad p^{\square} = p^7$

(g) $(p^{\square})^8 = p^{40}$ (h) $q^{13} \quad q^{\square} = q$

6. Calculate:

(a) $\dfrac{2^3}{2^2} + 3^0$ (b) $\dfrac{3^4}{3^3} - 3^0$

(c) $\dfrac{5^4}{5^2} + \dfrac{6^2}{6}$ (d) $\dfrac{7^7}{7^5} - \dfrac{5^9}{5^7}$

(e) $\dfrac{10^8}{10^5} - \dfrac{5^6}{5^3}$ (f) $\dfrac{4^{17}}{4^{14}} - \dfrac{4^{13}}{4^{11}}$

7. Fill in the missing numbers in each of the following expressions:

(a) $8^2 = 2^{\square}$ (b) $81^3 = 9^{\square} = 3^{\square}$

(c) $25^6 = 5^{\square}$ (d) $4^7 = 2^{\square}$

(e) $125^4 = 5^{\square}$ (f) $1000^6 = 10^{\square}$

(g) $81 = \square^4$ (h) $256 = \square^4 = \square^8$

8. Fill in the missing numbers in each of the following expressions:

(a) $8 \quad 4 = 2^{\square} \quad 2^{\square}$
$= 2^{\square}$

(b) $25 \quad 625 = 5^{\square} \quad 5^{\square}$
$= 5^{\square}$

(c) $\dfrac{243}{9} = \dfrac{3^{\square}}{3^{\square}}$
$= 3^{\square}$

(d) $\dfrac{128}{16} = \dfrac{2^{\square}}{2^{\square}}$
$= 2^{\square}$

9. Is each of the following statements true or false?

(a) $3^2 \quad 2^2 = 6^4$ (b) $5^4 \quad 2^3 = 10^7$

(c) $\dfrac{6^8}{2^8} = 3^8$ (d) $\dfrac{10^8}{5^6} = 2^2$

10. Copy and complete each expression:

(a) $(2^6 \quad 2^3)^4 = (2^{\square})^4 = 2^{\square}$ (b) $\left[\dfrac{3^6}{3^2}\right]^5 = (3^{\square})^5 = 3^{\square}$

(c) $\left[\dfrac{2^3 \quad 2^4}{2^7}\right]^4 = (2^{\square})^4 = 2^{\square}$ (d) $\left[\dfrac{3^2 \quad 9}{3^3}\right]^4 = (3^{\square})^4 = 3^{\square}$

(e) $\left[\dfrac{6^2 \quad 6^8}{6^3}\right]^4 = (6^{\square})^4 = 6^{\square}$ (f) $\left[\dfrac{7^8}{7^2 \quad 7^3}\right]^5 = (7^{\square})^5 = 7^{\square}$

Negative Exponents

In this section we practice working with negative exponents. From our work in the last section, we see that

$$a^2 \quad a^3 = a^{2-3} = a^{-1}$$

but we know that

$$a^2 \quad a^3 = \frac{a \quad a}{a \quad a \quad a} = \frac{1}{a}, \text{ a fraction}$$

so

$$a^{-1} = \frac{1}{a}$$

in the same way

$$a^{-2} = \frac{1}{a^2}$$

$$= \frac{1}{a \quad a}$$

$$a^{-3} = \frac{1}{a^3}$$

$$= \frac{1}{a \quad a \quad a}$$

and, in general,

$$a^{-n} = \frac{1}{a^n}$$

for positive integer values of n. The four rules on page 9 can now be used for any integers m and n, not just for positive values.

Example 1

Calculate, leaving your answers as fractions:

(a) 3^{-2} (b) $2^{-1} - 4^{-1}$ (c) 5^{-3}

Solution

(a) $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

(b) $2^{-1} - 4^{-1} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

(c) $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$

Example 2

Simplify:

(a) $\frac{6^7}{6^9}$ (b) $6^4 \quad 6^{-3}$ (c) $(10^2)^{-3}$

Solution

(a) $\frac{6^7}{6^9} = 6^{7-9} = 6^{-2} = \frac{1}{6^2} = \frac{1}{36}$

(b) $6^4 \quad 6^{-3} = 6^{4+(-3)} = 6^{4-3} = 6^1 = 6$

(c) $(10^2)^{-3} = 10^{-6} = \frac{1}{10^6} = \frac{1}{1\,000\,000}$

Skill Exercises: Negative Exponents

1. Write the following numbers as fractions without using any exponents:

(a) 4^{-1} (b) 2^{-3} (c) 10^{-3}

(d) 7^{-2} (e) 4^{-3} (f) 6^{-2}

2. Copy the following expressions and fill in the missing numbers:

(a) $\frac{1}{49} = \frac{1}{7^{\square}} = 7^{\square}$ (b) $\frac{1}{100} = \frac{1}{10^{\square}} = 10^{\square}$

(c) $\frac{1}{81} = \frac{1}{9^{\square}} = 9^{\square}$ (d) $\frac{1}{16} = \frac{1}{2^{\square}} = 2^{\square}$

(e) $\frac{1}{10\,000\,000} = \frac{1}{10^{\square}} = 10^{\square}$ (f) $\frac{1}{1024} = \frac{1}{2^{\square}} = 2^{\square}$

3. Calculate:

(a) $4^{-1} + 3^{-1}$ (b) $6^{-1} + 2^{-1}$ (c) $5^{-1} - 10^{-1}$

(d) $10^{-2} - 10^{-3}$ (e) $4^{-1} - 10^{-1}$ (f) $6^{-1} + 7^{-1}$

4. Simplify the following expressions giving your answers in the form of a number to a power:

(a) $4^7 \quad 4^{-6}$ (b) $5^7 \quad 5^{-3}$ (c) $\dfrac{7^4}{7^{-6}}$

(d) $(3^2)^{-4}$ (e) $(6^{-2})^{-3}$ (f) $8^4 \quad 8^{-9}$

(g) $\dfrac{7^2}{7^{-2}}$ (h) $\dfrac{8^9}{8^{-9}}$

5. Copy each of the following expressions and fill in the missing numbers:

(a) $\dfrac{1}{9} = 3^{\square}$ (b) $\dfrac{1}{100} = 10^{\square}$

(c) $\dfrac{1}{125} = 5^{\square}$ (d) $\dfrac{5}{5^4} = 5^{\square}$

(e) $\dfrac{6^2}{6^3} = 6^{\square}$ (f) $\dfrac{2^2}{2^{10}} = 2^{\square}$

6. Simplify the following expressions:

(a) $\dfrac{x^8}{x^3}$ (b) $\dfrac{x^7}{x^9}$ (c) $\dfrac{x^4}{x^8}$

(d) $(x^6)^{-4}$ (e) $\left[\dfrac{1}{x^2}\right]^4$ (f) $(x^{-8})^3$

7. Copy and complete the following statements:

(a) $0.1 = 10^{\square}$ (b) $0.25 = 2^{\square}$ (c) $0.0001 = 10^{\square}$

(d) $0.2 = 5^{\square}$ (e) $0.001 = 10^{\square}$ (f) $0.02 = 50^{\square}$

8. Copy the following expressions and fill in the missing numbers:

(a) $\dfrac{x^4}{x^{\square}} = x^2$ (b) $x^6 \quad x^{\square} = x^2$

(c) $x^9 \quad x^{\square} = x^2$ (d) $\dfrac{x^7}{x^{\square}} = x^{-2}$

(e) $\dfrac{x^3}{x^{\square}} = x^4$ (f) $(x^3)^{\square} = x^{-6}$

9. Copy the following expressions and fill in the missing numbers:

(a) $\dfrac{1}{8} = 2^{\square}$ (b) $\dfrac{1}{25} = 5^{\square}$

(c) $\dfrac{1}{81} = 9^{\square}$ (d) $\dfrac{1}{10\,000} = 10^{\square}$

Fractional Exponents

Exponents that are fractions are used to represent square roots, cube roots and other roots of numbers.

$a^{\frac{1}{2}}$	$=$	$\sqrt{a}$	for example,	$9^{\frac{1}{2}}$	$=$	3
$a^{\frac{1}{3}}$	$=$	$\sqrt[3]{a}$	for example,	$8^{\frac{1}{3}}$	$=$	2
$a^{\frac{1}{4}}$	$=$	$\sqrt[4]{a}$	for example,	$625^{\frac{1}{4}}$	$=$	5

The rule is:

$$a^{\frac{1}{n}} = \sqrt[n]{a}$$

Example 1

Calculate:

(a) $81^{\frac{1}{2}}$ (b) $1000^{\frac{1}{3}}$ (c) $4^{-\frac{1}{2}}$

Solution

(a) $81^{\frac{1}{2}} = \sqrt{81}$
$= 9$

(b) $1000^{\frac{1}{3}} = \sqrt[3]{1000}$
$= 10$

(c) $4^{-\frac{1}{2}} = \dfrac{1}{4^{\frac{1}{2}}}$
$= \dfrac{1}{\sqrt{4}}$
$= \dfrac{1}{2}$

Skill Exercises: Fractional Exponents

1. Calculate:

(a) $49^{\frac{1}{2}}$ (b) $64^{\frac{1}{2}}$ (c) $16^{\frac{1}{2}}$
(d) $81^{-\frac{1}{2}}$ (e) $100^{-\frac{1}{2}}$ (f) $25^{-\frac{1}{2}}$
(g) $9^{\frac{1}{2}}$ (h) $36^{-\frac{1}{2}}$ (i) $144^{\frac{1}{2}}$

2. Calculate:

(a) $8^{\frac{1}{3}}$ (b) $8^{-\frac{1}{3}}$ (c) $125^{\frac{1}{3}}$
(d) $64^{-\frac{1}{3}}$ (e) $216^{\frac{1}{3}}$ (f) $1\,000\,000^{-\frac{1}{3}}$

3. Calculate:

(a) $32^{\frac{1}{5}}$ (b) $64^{-\frac{1}{2}}$ (c) $10\,000^{\frac{1}{4}}$
(d) $81^{-\frac{1}{4}}$ (e) $625^{\frac{1}{4}}$ (f) $100\,000^{-\frac{1}{5}}$

4. Calculate:

(a) $\left[\dfrac{4 \times 8}{2}\right]^{\frac{1}{2}}$ (b) $\left[\dfrac{9 \times 27}{3}\right]^{\frac{1}{4}}$ (c) $\left[\dfrac{125 \times 5}{25}\right]^{\frac{1}{2}}$

5. Is each of the following statements true or false?

(a) $16^{\frac{1}{2}} = 8$ (b) $16^{\frac{1}{4}} = 2$ (c) $81^{\frac{1}{3}} = 9$

Calculating Exponents with a Calculator

1. To square a number use $\boxed{x^2}$

e.g. $3^2 \longrightarrow \boxed{3}\ \boxed{x^2}\ \boxed{=} \longrightarrow 9$

2. To find the square root of a number use $\boxed{\sqrt{\ }}$

e.g. $\sqrt{9} \longrightarrow \boxed{\sqrt{\ }}\ \boxed{9}\ \boxed{=} \longrightarrow 3$

3. To calculate expressions with other exponents use $\boxed{y^x}$

e.g. $4^3 \longrightarrow \boxed{4}\ \boxed{y^x}\ \boxed{3}\ \boxed{=} \longrightarrow 64$

$5^{-2} \longrightarrow \boxed{5}\ \boxed{y^x}\ \boxed{+/-}\ \boxed{2}\ \boxed{=} \longrightarrow 0.04$

4. To calculate expressions with fractional exponents use the fraction key $\boxed{a^b/c}$

e.g. $4^{\frac{1}{2}} \longrightarrow \boxed{4}\ \boxed{y^x}\ \boxed{1}\ \boxed{a^b/c}\ \boxed{2}\ \boxed{=} \longrightarrow 2$

$27^{\frac{2}{3}} \longrightarrow \boxed{2}\ \boxed{7}\ \boxed{y^x}\ \boxed{2}\ \boxed{a^b/c}\ \boxed{3}\ \boxed{=} \longrightarrow 9$

$25^{-\frac{1}{2}} \longrightarrow \boxed{2}\ \boxed{5}\ \boxed{y^x}\ \boxed{+/-}\ \boxed{1}\ \boxed{a^b/c}\ \boxed{2}\ \boxed{=} \longrightarrow 0.2$

5. To calculate roots of numbers use the root key $\overset{\sqrt[x]{\ }}{\boxed{y^x}}$

This is a second function operation so the $\boxed{2_{nd}F}$ key will have to be pressed first.

e.g. $\sqrt[3]{27} \longrightarrow \boxed{3}\ \boxed{2_{nd}F}\ \boxed{\sqrt[x]{\ }}\ \boxed{2}\ \boxed{7}\ \boxed{=} \longrightarrow 3$

$\sqrt[4]{256} \longrightarrow \boxed{4}\ \boxed{2_{nd}F}\ \boxed{\sqrt[x]{\ }}\ \boxed{2}\ \boxed{5}\ \boxed{6}\ \boxed{=} \longrightarrow 4$

Calculator Skills: Exponents

Use a calculator to evaluate:

1. (a) 5^2 (b) 6^2 (c) 1^2 (d) 15^2

2 (a) $\sqrt{36}$ (b) $\sqrt{144}$ (c) $\sqrt{256}$ (d) $\sqrt{10\,000}$

3. (a) 6^3 (b) 10^3 (c) 2^{-2} (d) 10^{-3}

4. (a) $121^{\frac{1}{2}}$ (b) $64^{\frac{2}{3}}$ (c) $100^{-\frac{1}{2}}$ (d) $8^{-\frac{2}{3}}$

5. (a) $\sqrt[3]{64}$ (b) $\sqrt[3]{216}$ (c) $\sqrt[4]{81}$ (d) $\sqrt[5]{32}$

Section 1.3 Writing In Standard Form

What is Standard Form?

Standard form is a convenient way of writing very large or very small numbers. It is used on a scientific calculator when a number is too large or too small to be displayed on the screen.

Before using standard form, we revise multiplying and dividing by powers of 10.

Example 1

Calculate:

(a) 3×10^4 (b) 3.27×10^3

(c) $3 \div 10^2$ (d) $4.32 \div 10^4$

Solution

(a) $3 \times 10^4 = 3 \times 10\,000$
$= 30\,000$

(b) $3.27 \times 10^3 = 3.27 \times 1000$
$= 3270$

(c) $3 \div 10^2 = \dfrac{3}{100}$
$= 0.03$

(d) $4.32 \div 10^4 = \dfrac{4.32}{10\,000}$
$= \dfrac{432}{1\,000\,000}$
$= 0.000432$

These examples lead to the approach used for standard form, which is a reversal of the approach used in Example 1.

> In standard form, numbers are written as
>
> $a \times 10^n$
>
> Where $1 \leq a < 10$ and n is an integer.

Example 2

Write the following numbers in standard form:

(a) 5720 (b) 7.4

(c) 473 000 (d) 6 000 000

(e) 0.09 (f) 0.000621

Solution

(a) $\begin{aligned} 5720 &= 5.72 \quad 1000 \\ &= 5.72 \quad 10^3 \end{aligned}$

(b) $\begin{aligned} 7.4 &= 7.4 \quad 1 \\ &= 7.4 \quad 10^0 \end{aligned}$

(c) $\begin{aligned} 473\,000 &= 4.73 \quad 100\,000 \\ &= 4.73 \quad 10^5 \end{aligned}$

(d) $\begin{aligned} 6\,000\,000 &= 6 \quad 1\,000\,000 \\ &= 6 \quad 10^6 \end{aligned}$

(e) $\begin{aligned} 0.09 &= \frac{9}{100} \\ &= 9 \quad 10^2 \\ &= 9 \quad 10^{-2} \end{aligned}$

(f) $\begin{aligned} 0.000621 &= \frac{6.21}{10\,000} \\ &= \frac{6.21}{10^4} \\ &= 6.21 \quad 10^{-4} \end{aligned}$

Example 3

Calculate:

(a) $(3 \quad 10^6) \quad (4 \quad 10^3)$

(b) $(6 \quad 10^7) \quad (5 \quad 10^{-2})$

(c) $(3 \quad 10^4) + (2 \quad 10^5)$

Solution

(a) $\begin{aligned} (3 \quad 10^6) \quad (4 \quad 10^3) &= (3 \quad 4) \quad (10^6 \quad 10^3) \\ &= 12 \quad 10^9 \\ &= 1.2 \quad 10^1 \quad 10^9 \\ &= 1.2 \quad 10^{10} \end{aligned}$

(b) $\begin{aligned} (6 \quad 10^7) \quad (5 \quad 10^{-2}) &= (6 \quad 5) \quad (10^7 \quad 10^{-2}) \\ &= 1.2 \quad 10^9 \end{aligned}$

(c) $\begin{aligned} (3 \quad 10^4) + (2 \quad 10^5) &= 30\,000 + 200\,000 \\ &= 230\,000 \\ &= 2.3 \quad 10^5 \end{aligned}$

Skill Exercises: Standard Form

1. Calculate:

(a) 6.21 1000 (b) 8 10^3 (c) 4.2 10^2

(d) 3 1000 (e) 6 10^2 (f) 3.2 10^3

(g) 6 10^{-3} (h) 9.2 10^{-1} (i) 3.6 10^{-2}

2. Write each of the following numbers in standard form:

(a) 200 (b) 8000 (c) 9 000 000

(d) 62 000 (e) 840 000 (f) 12 000 000 000

(g) 61 800 000 000 (h) 3 240 000

3. Convert each of the following numbers from standard form to the normal decimal notation:

(a) 3 10^4 (b) 3.6 10^4 (c) 8.2 10^3

(d) 3.1 10^2 (e) 1.6 10^4 (f) 1.72 x 10^5

(g) 6.83 10^4 (h) 1.25 10^6 (i) 9.17 10^3

4. Write each of the following numbers in standard form:

(a) 0.0004 (b) 0.008 (c) 0.142

(d) 0.0032 (e) 0.00199 (f) 0.000000062

(g) 0.0000097 (h) 0.00000000000021

5. Convert the following numbers from standard form to the normal decimal format:

(a) 6 10^{-2} (b) 7 10^{-1} (c) 1.8 10^{-3}

(d) 4 10^{-3} (e) 6.2 10^{-3} (f) 9.81 10^{-4}

(g) 6.67 10^{-1} (h) 3.86 10^{-5} (i) 9.27 10^{-7}

6. Calculate: (Don't use a calculator)

(a) (4 10^4) (2 10^5) (b) (2 10^6) (3 10^5)

(c) (6 10^4) (8 10^{-9}) (d) (3 10^{-8}) (7 10^{-4})

(e) (6.1 10^6) (2 10^{-5}) (f) (3.2 10^{-5}) (4 10^{-9})

7. Calculate: (Don't use a calculator)

(a) (9 10^7) (3 10^4) (b) (8 10^5) (2 10^{-2})

(c) (6 10^{-2}) (2 10^{-3}) (d) (6 10^4) (3 10^{-6})

(e) (4.8 10^{12}) (1.2 10^3) (f) (3.6 10^8) (9 10^3)

Using a Calculator with Standard Form

To enter a number in standard form, use the $\boxed{\text{EXP}}$ key.

e.g. 3.2 10^4 $\longrightarrow$ $\boxed{3}\ \boxed{\cdot}\ \boxed{2}\ \boxed{\text{EXP}}\ \boxed{4}\ \boxed{=}$ $\longrightarrow$ 32 000

e.g. (3 10^4) (4 10^3) $\longrightarrow$

$\boxed{(}\ \boxed{3}\ \boxed{\text{EXP}}\ \boxed{4}\ \boxed{)}\ \boxed{\text{X}}\ \boxed{(}\ \boxed{4}\ \boxed{\text{EXP}}\ \boxed{3}\ \boxed{)}$

$\boxed{=}$ $\longrightarrow$ 120 000 000

If the answer is large, the calculator will display it in standard form.

e.g. (2 10^{15}) (3 10^4) $\longrightarrow$ 6 10^{19}

Some calculators may show this as: 6E19

Calculator Skills: Standard Form

Use a calculator to work out the following. Give your answers in both normal and standard form.

1. (a) (6 10^5) + (3 10^6) (b) (6 10^2) + (9 10^3)

   (c) (6 10^5) – (1 10^4) (d) (8 10^{-2}) + (9 10^{-3})

   (e) (6 10^{-4}) + (8 10^{-3}) (f) (6 10^{-4}) – (3 10^{-5})

2. Use a calculator to determine:

   (a) (3.4 10^6) (2.1 10^4) (b) (6 10^{21}) (8.2 10^{-11})

   (c) (3.6 10^5) (4.5 10^7) (d) (8.2 10^{11}) (4 10^{-8})

   (e) (1.92 10^6) (3.2 10^{-11}) (f) (6.2 10^{14})^3

3. (a) Which of these statements is true?

   (i) 4 10^3 is a larger number than 4^3.

   (ii) 4 10^3 is the same size as 4^3.

   (iii) 4 10^3 is a smaller number than 4^3.

   Explain your answer.

(b) One of the numbers below has the same value as $3.6 \quad 10^4$.
Write down the number.

36^3 $\qquad$ $(3.6 \quad 10)^4$ $\qquad$ 36^4 $\qquad$ $0.36 \quad 10^3$ $\qquad$ $0.36 \quad 10^5$

(c) One of the numbers below has the same value as $2.5 \quad 10^{-3}$.
Write down the number.

$25 \quad 10^{-4}$ $\qquad$ $2.5 \quad 10^3$ $\qquad$ $-2.5 \quad 10^3$ $\qquad$ 0.00025 $\qquad$ 2500

(d) $(2 \quad 10^2) \quad (2 \quad 10^2)$ can be written more simply as $4 \quad 10^4$.
Simplify the following:

(i) $(3 \quad 10^2) \quad (2 \quad 10^{-2})$

(ii) $\dfrac{6 \quad 10^8}{2 \quad 10^4}$

Unit 2: ALGEBRA

In this unit you will be:

2.1 Applying the Laws of Exponents

- Exponents in Algebra.

2.2. Simplifying Algebraic Expressions

- Substitution into Algebraic Expressions.
- Collecting Like Terms.

2.3 Solving Linear Equations

- Solving Linear Equations (x on one side).
- Solving Linear Equations (x on both sides).
- Solving Linear Equations (with brackets).

2.4 Solving Linear Inequalities

- Writing Inequalities.
- Showing Inequalities on a Number Line.
- Solving Linear Inequalities.

2.5 Writing Linear Equations

Section 2.1 Applying The Laws Of Exponents In Algebra

Exponents in Algebra

Algebra is a branch of mathematics in which numbers are replaced with letters (called variables).

e.g. $4x^3 + 2x^2 + 3y$, $2a + 4bc$ or $x^2 + y + xy$

are examples of algebraic expressions.

The laws of exponents that were used in Number – Part 1 are also used in Algebra.

$$a^m \quad a^n = a^{m+n} \qquad \text{and} \qquad \frac{a^m}{a^n} = a^{n-m}$$

$$(a^m)^n = a^{m \quad n} \qquad\qquad a^0 = 1$$

Examples

Simplify each of the following expressions:

(a) $x^6 \quad x^7 = x^{6+7}$
$= x^{13}$

(b) $\dfrac{y^{14}}{y^{10}} = y^{14-10}$
$= y^4$

(c) $(z^2)^4 = z^{2 \quad 4}$
$= z^8$

(d) $\dfrac{x^5}{x^2 \quad x^3} = \dfrac{x^5}{x^5}$
$= x^{5-5}$
$= x^0$
$= 1$

(e) $3x^2 \quad 2x^4 = (3 \quad 2)x^{2+4}$
$= 6x^6$

(f) $\dfrac{10x^4}{5x^2} = \left[\dfrac{10}{5}\right] x^{4-2}$
$= 2x^2$

(g) $(3x^2)^3 = (3^3)x^{2 \quad 3}$
$= 27x^6$

Skill Exercises: Exponents in Algebra

1. Simplify each of the following expressions:

(a) $a^3 \quad a^2 =$ (b) $a^4 \quad a^6 =$ (c) $x^2 \quad x^7 =$

(d) $x^4 \quad x^2 =$ (e) $y^3 \quad y^0 =$ (f) $p^7 \quad p^4 =$

(g) $q^6 \quad q^3 =$ (h) $x^7 \quad x =$ (i) $b^4 \quad b =$

(j) $\frac{b^6}{b^0} =$ (k) $\frac{c^7}{c^4} =$ (l) $\frac{x^8}{x^3} =$

(m) $\frac{y^3}{y} =$ (n) $\frac{x^4}{x^4} =$ (o) $x^2 \quad x^3 \quad x^3 =$

(p) $\frac{p^2 \quad p^7}{p^5} =$ (q) $\frac{x^{10}}{x^2 \quad x^5} =$ (r) $\frac{y^3}{y^2} \quad \frac{y^7}{y^4} =$

(s) $\frac{x^2 \quad x^3}{x^5} =$ (t) $\frac{x^7}{x^3} \quad \frac{x}{x^4} =$ (u) $\frac{x^8 \quad x^4}{x^0} =$

(v) $(x^2)^4 =$ (w) $(x^3)^5 =$ (x) $(x^2 \quad x^7)^6 =$

2. 243 can be written as 3^5.

Find the values of p and q in the following:

(a) $64 = 4^p$ (b) $5^q = 1$

3. Simplify the following:

(a) $2x^4 \quad 4x^3$ (b) $4x^2 \quad 8x^5$ (c) $3x^4 \quad x^2$

(d) $6x^4 \quad 2x^5$

4. Simplify the following:

(a) $\frac{6x^3}{2x}$ (b) $\frac{20a^5}{4a^3}$ (c) $\frac{3x^2 \quad 4x^4}{6x^3}$ (d) $\frac{2p^3 \quad 2p^2 \quad p}{2p^5}$

5. Simplify the following:

(a) $(3x^3)^2$ (b) $(4a^4)^2$ (c) $\frac{(2p^3)^4}{8p^6}$ (d) $\frac{4q^2 \quad 6q^5}{(2q^3)^3}$

Section 2.2 Simplifying Algebraic Expressions

Substitution into Algebraic Expressions

Substitution means to replace the letters in an algebraic expression with numbers.

Example

If $a = 4$, $b = 7$ and $c = 3$, calculate:

(a) $6 + b$ (b) $2a + b$ (c) ab (d) $a(b - c)$

Solution

(a) $6 + b = 6 + 7$

$= 13$

(b) $2a + b = 2 \quad 4 + 7$ since $\boxed{2a = 2 \quad a}$

$= 8 + 7$

$= 15$

(c) $ab = 4 \quad 7$ since $\boxed{ab = a \quad b}$

$= 28$

(d) $a(b - c) = 4 \quad (7 - 3)$ since $\boxed{a(b - c) = (a \quad (b - c)}$

$= 4 \quad 4$

$= 16$

Skill Exercises: Substitution

1. If $a = 2$, $b = 6$, $c = 10$ and $d = 3$, calculate:

(a) $a + b$ (b) $c - b$ (c) $d + 7$
(d) $3a + d$ (e) $4a$ (f) ad
(g) $3b$ (h) $2c$ (i) $3c - b$
(j) $6a + b$ (k) $3a + 2b$ (l) $4a - d$

2. If $a = 3$, $b = {}^-1$, $c = 2$ and $d = {}^-4$, calculate:

(a) $a - b$ (b) $a + d$ (c) $b + d$
(d) $b - d$ (e) $3d$ (f) $a + b$
(g) $c - d$ (h) $2c + d$ (i) $3a - d$
(j) $2d + 3c$ (k) $4a - 2d$ (l) $5a + 3d$

3. If $a = 7$, $b = 5$, $c = {}^{-}3$ and $d = 4$, calculate:

(a) $2(a + b)$	(b) $4(a - b)$	(c) $6(a - d)$
(d) $2(a + c)$	(e) $5(b - c)$	(f) $5(d - c)$
(g) $a(b + c)$	(h) $d(b + a)$	(i) $c(b - a)$
(j) $a(2b - c)$	(k) $d(2a - 3b)$	(l) $c(d - 2)$

4. Use the formula $s = \frac{1}{2}(u + v)t$ to find s, when $u = 10$, $v = 20$ and $t = 4$.

5. Use the formula $v = u + at$ to find v, if $u = 20$, $a = {}^{-}2$ and $t = 7$.

6. If $x = 4$ and $y = 3$, find the values of:

(a) $2x^2$	(b) $x^2 + y^2$	(c) $2x - y$
(d) $2xy$	(e) $\dfrac{x + 2}{y}$	(f) $2y^2$
(g) y^3	(h) $4y - 3x$	(i) $\dfrac{y^2 + 1}{x}$
(j) $\sqrt{x + 4y}$		

7. If $a = 5$, $b = 3$ and $c = 1$, find the values of x if:

(a) $x = 4a + b$	(b) $x = a^2 + b^2$	(c) $x = 2a^2$
(d) $x = 2a - 3b - c$	(e) $x = \dfrac{a}{c}$	

Problem Solving Skills: Substitution

1. If distance = speed time, what is distance when speed = 70 and time = 3?

2. If paint = $\dfrac{\text{area}}{18}$, what is paint when area = 45?

3. If amount = principal + interest, what is amount when principal = 800 and interest = 80?

4. If weight = 6 (length)^2, what is weight when length = 5?

5. If radius = $\sqrt{\dfrac{7 \times \text{area}}{22}}$, what is radius when area = 154?

Collecting Like Terms

Algebraic expressions can be simplified by collecting like terms together.

Example 1

6 apples + 3 bananas + 2 apples + 4 bananas = 8 apples + 7 bananas

$$6a + 3b + 2a + 4b = 8a + 7b$$

Example 2

Simplify where possible:

(a) $2x + 4x$ (b) $5p + 7q - 3p + 2q$

(c) $y + 8y - 5y$ (d) $3t + 4s$

(e) $3(a + 4b)$ (f) $3(g + 4h) + 2(3g - h)$

Solution

(a) $2x + 4x$
$= 2 \quad x + 4 \quad x$
$= (x + x) + (x + x + x + x)$
$= 6 \quad x$
$= 6x$

(b) $5p + 7q - 3p + 2q$
$= 5p - 3p + 7q + 2q$
$= (5 - 3)p + (7 + 2)q$
$= 2p + 9q$

(c) $y + 8y - 5y$
$= 1y + 8y - 5y$
$= (1 + 8 - 5)y$
$= 4y$

(d) $3t + 4s$ cannot be simplified.

(e) $3(a + 4b)$ $= 3a + 12b$

(f) $3(g + 4h) + 2(3g - h)$ $= 3g + 12h + 6g - 2h$
$= 9g + 10h$

Skill Exercises: Collecting Like Terms

1. Simplify, where possible:

(a) $2a + 3a$ (b) $5b + 8b$

(c) $6c - 4c$ (d) $5d + 4d + 7d$

(e) $6e + 9e - 5e$ (f) $8f + 6f - 13f$

(g) $9g + 7g - 8g - 2g - 6g$ (h) $5p + 2h$

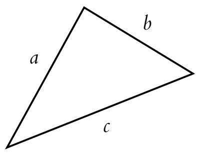
(i) $3a + 4b - 2a$ (j) $6x + 3y - 2x - y$

(k) $8t - 6t + 7s - 2s$

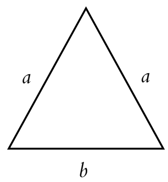
(l) $11m + 3n - 5p + 2q - 2n + 9q - 8m + 14p$

2. Write down the formula for the perimeter of each of these shapes:

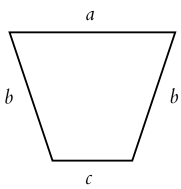
(a)



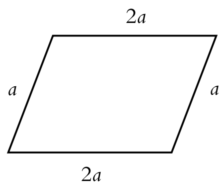
(b)



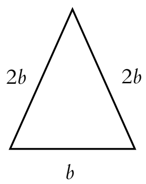
(c)



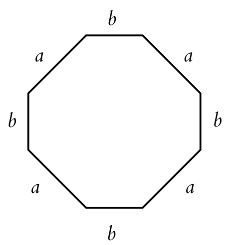
(d)



(e)



(f)



3. Remove the brackets in these expressions:

   (a) $2(3a - 5)$ (b) $3(6b + 5c)$

   (c) $4d(d + 1)$ (d) $5e(e^2 - e + 2)$

   (e) $3(f^2 - 18f + 4)$

4. Simplify:

   (a) $2(a + 2b) + (a - b)$ (b) $4(c - d) - 3(c + 2d)$

   (c) $3(2e + f) + 2(e - 2f)$ (d) $(g - h) - 4(g + 2h)$

   (e) $2j + 3k - (j - 3k)$ (f) $5(p - 2q - r) + 3(p - q + 2r)$

   (g) $3(s + 8) - 4(2s - 5)$ (h) $x(x - 4) + 3(x - 2)$

   (i) $x(2x + 3) - 4(3x - 1)$ (j) $x(x^2 + 1) - x^2(x + 1)$

Problem Solving Skills: Writing Formulae

1. Alofa asks her friend to think of a number, multiply it by 2 and then add 5. If the number her friend starts with is x, write down a formula for the number her friend gets.

2. A bus driver hires his bus at a fixed charge of \$50, plus \$2 for every kilometre travelled. Write down the formula for the cost of hiring the bus when travelling x kilometres.

3. A taxi driver charges passengers \$1 plus 50c per kilometre(s). Write down a formula for the cost of travelling x kilometres.

Section 2.3 Solving Linear Equations

Solving Linear Equations (x on one side)

In a linear equation the unknown variable is to the power 1.

e.g. $x + 7 = 5$ is a linear equation (x is to the power 1)

$x^2 + 7 = 5$ is not a linear equation (x is to the power 2)

Solving a linear equation means finding the unknown value. To solve a linear equation, reorganise it so that the unknown value is by itself on the left hand side of the 'equals' sign.

> An equation contains an 'equals' sign. When solving an equation, whatever is done to the left hand side must also be done to the right hand side.

Example

Solve these equations:

(a) $x + 2 = 8$ (b) $x - 4 = 3$ (c) $3x = 12$

(d) $\frac{x}{2} = 7$ (e) $2x + 5 = 11$ (f) $3 - 2x = 7$

Solution

(a) To solve this equation, subtract 2 from each side of the equation:

$$\begin{aligned} x + 2 &= 8 \\ x + 2 - 2 &= 8 - 2 \\ x &= 6 \end{aligned}$$

(b) To solve this equation, add 4 to both sides of the equation:

$$\begin{aligned} x - 4 &= 3 \\ x - 4 + 4 &= 3 + 4 \\ x &= 7 \end{aligned}$$

(c) To solve this equation, divide both sides of the equation by 3:

$$\begin{aligned} 3x &= 12 \\ \frac{3x}{3} &= \frac{12}{3} \\ x &= 4 \end{aligned}$$

(d) To solve this equation, multiply both sides of the equation by 2:

$$\frac{x}{2} = 7$$

$$2 \quad \frac{x}{2} = 2 \quad 7$$

$$x = 14$$

(e) The equation must be solved in two stages.

First, subtract 5 from both sides:

$$2x + 5 = 11$$

$$2x + 5 - 5 = 11 - 5$$

$$2x = 6$$

Then, divide both sides of the equation by 2:

$$\frac{2x}{2} = \frac{6}{2}$$

$$x = 3$$

(f) First, subtract 3 from both sides:

$$3 - 2x = 7$$

$$3 - 2x - 3 = 7 - 3$$

$$^-2x = 4$$

Then divide both sides by ($^-2$):

$$\frac{^-2x}{^-2} = \frac{4}{^-2}$$

$$x = {}^-2$$

Skill Exercises: Solving Linear Equations (*x* on one side)

1. Solve these equations:

(a) $x + 2 = 8$ (b) $x + 5 = 11$ (c) $x - 6 = 2$

(d) $x - 4 = 3$ (e) $2x = 18$ (f) $3x = 24$

(g) $\frac{x}{6} = 4$ (h) $\frac{x}{5} = 9$ (i) $6x = 54$

(j) $x + 12 = 10$ (k) $x + 5 = 3$ (l) $x - 22 = {}^-4$

(m) $\frac{x}{7} = {}^-2$ (n) $10x = 0$ (o) $\frac{x}{2} + 4 = 5$

2. Solve these equations:

(a) $2x + 4 = 14$ (b) $3x + 7 = 25$ (c) $4x + 2 = 22$

(d) $6x - 4 = 26$ (e) $5x - 3 = 32$ (f) $11x - 4 = 29$

(g) $3x + 4 = 25$ (h) $5x - 8 = 37$ (i) $6x + 7 = 31$

(j) $3x + 11 = 5$ (k) $6x + 2 = {}^-10$ (l) $7x + 44 = 2$

3. Solve these equations, giving your answers as fractions or mixed numbers:

   (a) $3x = 4$ (b) $5x = 7$ (c) $2x + 8 = 13$

   (d) $8x + 2 = 5$ (e) $2x + 6 = 9$ (f) $4x - 7 = 10$

Solving Linear Equations (x on both sides)

Example

Solve these equations:

(a) $3x + 2 = 4x - 3$ (b) $2x + 7 = 8x - 11$

Solution

These equations contain x on both sides. The first step is to change them so that x is on only one side of the equation. Choose the side which has the most x; here, the right hand side.

(a) Subtract $3x$ from both sides of the equation:

$$\begin{aligned} 3x + 2 &= 4x - 3 \\ 3x + 2 - 3x &= 4x - 3 - 3x \\ 2 &= x - 3 \end{aligned}$$

Then add 3 to both sides of the equation:

$$\begin{aligned} 2 &= x - 3 \\ 2 + 3 &= x - 3 + 3 \\ 5 &= x \\ \text{so} \qquad x &= 5 \end{aligned}$$

(b) First, subtract $2x$ from both sides of the equation:

$$\begin{aligned} 2x + 7 &= 8x - 11 \\ 2x + 7 - 2x &= 8x - 11 - 2x \\ 7 &= 6x - 11 \end{aligned}$$

Next, add 11 to both sides of the equation:

$$\begin{aligned} 7 + 11 &= 6x - 11 + 11 \\ 18 &= 6x \end{aligned}$$

Then divide both sides by 6:

$$\frac{18}{6} = \frac{6x}{6}$$

$$3 = x$$

so $$x = 3$$

Skill Exercises: Solving Linear Equations: (*x* on both sides)

1. Solve these equations:

(a) $x + 2 = 2x - 1$ (b) $8x - 1 = 4x + 11$

(c) $5x + 2 + 6x - 4$ (d) $11x - 4 = 2x + 23$

(e) $5x + 1 = 6x - 8$ (f) $3x + 2 + 5x = x + 44$

(g) $6x + 2 - 2x = x + 23$ (h) $2x - 3 = 6x + x - 58$

(i) $3x + 2 = x - 8$ (j) $4x - 2 = 2x - 8$

(k) $3x + 82 = 10x + 12$ (l) $6x - 10 = 2x - 14$

Solving Linear Equations (with brackets)

Example

Solve:

(a) $5(x - 3) = 35$ (b) $6(x + 7) = 50$

Solution

(a) $5(x - 3) = 35$

Expanding brackets gives: $5x - 15 = 35$

Adding 15 to both sides gives: $5x = 50$

Dividing by 5 gives: $x = 10$

(b) $6(x + 7) = 50$

Expanding brackets gives: $6x + 42 = 50$

Subtracting 42 from both sides gives: $6x = 8$

Dividing by 6 gives: $x = \frac{8}{6}$

$= 1\frac{1}{3}$

Skill Exercises: Solving Linear Equations (with brackets)

1. Solve these equations:

   (a) $2(x + 6) = 14$ (b) $5(x - 8) = 40$

   (c) $3(x + 5) = 12$ (d) $7(x + 4) = 42$

   (e) $2(x + 7) = 19$ (f) $3(x - 4) = 11$

   (g) $5(x - 4) = 12$ (h) $10(x + 7) = 82$

2. Solve these equations:

   (a) $5(2x - 7) = 8$ (b) $3(3x + 6) = 27$

   (c) $3(2x + 1) = 30$ (d) $8(2x - 12) = 24$

3. Solve the following equations:

   (a) $4(7 - x) = 20$ (b) $3(9 - x) = 15$

   (c) $6(5 - 2x) = 18$ (d) $5(7 - 3x) = 20$

   (e) $2(10 - 3x) = 17$ (f) $6(9 - 5x) = 4$

4. Solve the following equations:

   (a) $2(x + 1) = 6(x - 3)$ (b) $3(x + 4) = 11x$

   (c) $5(x + 4) = 2(10x + 1)$ (d) $4(7 - x) = 5(x + 2)$

Section 2.4 Solving Linear Inequalities

An inequality is a mathematical sentence that states that one quantity is greater than or less than another in value.

Writing Inequalities

$<$ is the symbol for 'is less than', so $x < 4$ means 'x is less than 4'.

$>$ is the symbol for 'is greater than'.

$\leq$ is the symbol for 'is less than or equal to'.

$\geq$ is the symbol for 'is greater than or equal to', so $x \geq 3$ means that 'x is greater than or equal to 3'.

Example

If x is an integer, what are the possible values of x if $-1 \le x < 5$?

x is greater than or equal to -1, and x is less than 5.

So the possible values of x are $-1, 0, 1, 2, 3, 4$.

Skill Exercises: Writing Inequalities

1. Describe these statements in words:

   (a) $x > 7$ (b) $x \le 8$ (c) $x < 1$

   (d) $1 < x < 4$ (e) $x \ge -5$

2. Write these statements as inequalities:

   (a) x is less than 6

   (b) x is greater than or equal to -2

   (c) x is greater than 0

   (d) x is less than 10 but greater than -3

   (e) x is less than or equal to 5

3. If x is an integer, what are the possible values of x if:

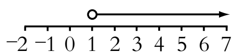
   (a) $3 < x < 7$ (b) $4 \le x < 6$ (c) $-2 \le x \le 2$

   (d) $-8 < x < -4$ (e) $0 \le x \le 5$ (f) $5 > x > 1$

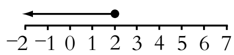
Showing Inequalities on a Number Line

Example

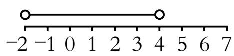
$x > 1$



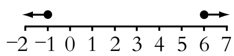
$x \le 2$



$-2 < x < 4$



$x \le -1$ or $x \ge 6$



We have used the symbol ● if the end point is included and the symbol ○ if the end point is not included.

Skill Exercises: Showing Inequalities on a Number Line

1. Show these inequalities on a number line. Draw a separate number line for each part, labeling each line from $^{-}4$ to 4.

   (a) $x > {}^{-}3$ (b) $x < {}^{-}1$ (c) $x \geq 0$

   (d) $x \leq 3$ (e) $^{-}2 < x < {}^{-}1$ (f) $^{-}3 \leq x \leq 4$

   (g) $x < {}^{-}3$ or $x > 2$ (h) $x \leq 1$ or $x \geq 2$

2. If x is an integer such that $^{-}4 \leq x \leq 4$, write down the possible values for x, for the inequalities of question 1.

Solving Linear Inequalities

To solve simple inequalities, use the same methods as for solving simple equations.

You can add equal numbers to both sides.

You can subtract equal numbers from both sides.

You can multiply both sides by the same positive number.

You can divide both sides by the same positive number.

If you multiply or divide both sides by a negative number, the inequality sign must be reversed at the same time.

Examples

1. Find the values of x which satisfy the inequality $13x - 20 > 6x + 8$.

	$13x - 20$	$>$	$6x + 8$
subtract $6x$ from both sides	$7x - 20$	$>$	8
add 20 to both sides	$7x$	$>$	28
divide both sides by 7	x	$>$	4

2. Find the values of x which satisfy the inequality $8 - 3x \geq 14$.

	$8 - 3x$	$\geq$	14
subtract 8 from both sides	$^{-}3x$	$\geq$	6
divide both sides by $^{-}3$	x	$\leq$	$^{-}2$
reverse the inequality sign			

Skill Exercises: Solving Linear Inequalities

Find the values of x which satisfy these inequalities.

(a) $6(x - 7) < 6$ (b) $x - 1 > 2x + 5$ (c) $5 - x \geq 6 - 3x$

(d) $\frac{x}{2} - 8 \leq -10$ (e) $12 - 2x < 0$ (f) $5(x + 1) \leq x + 8$

(g) $3(x - 4) < 5(x - 7)$ (h) $\frac{x-2}{3} \geq -1$ (i) $12x - 5 > 15 - 8x$

(j) $3(2x - 1) + 2(x + 1) \leq 39$

Section 2.5 Writing Linear Equations

Example

Fofoga thinks of a number and adds 7 to it. She then multiplies her answer by 4 and gets 64.

(a) Write down an equation that can be used to calculate the number with which Fofoga started.

(b) Solve your equation to give the number.

Solution

(a) Start with x

Add 7 to give $x + 7$

Multiply by 4 to give $4(x + 7)$

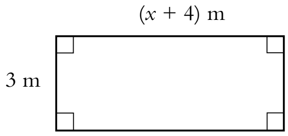
This expression equals 64, so the equation is $4(x + 7) = 64$

(b)

	$4(x + 7)$	$=$	64
Expanding brackets gives	$4x + 28$	$=$	64
Subtracting 28 from both sides gives	$4x$	$=$	36
Dividing by 4 gives	x	$=$	$\frac{36}{4}$
	x	$=$	9

Skill Exercises: Writing Linear Equations

1. A rectangle has sides of length 3 m and $(x + 4)$ m.
   Find the value of x, if the area of the rectangle is 18 m^2.



2. Feleti chooses a number, adds 7, multiplies the result by 5 and gets the answer 55.

   (a) If x is the number Feleti first chose, write down an equation that can be used to determine the number.

   (b) Solve the equation to determine the value of x.

3. The following flow chart is used to form an equation:

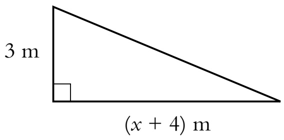
$$x \longrightarrow \boxed{+\ 6} \longrightarrow \boxed{\quad 4} \longrightarrow 17$$

   (a) Write down the equation.

   (b) Solve the equation to find the value of x.

4. Lauulu thinks of a number, subtracts it from 11 and then multiplies his answer by 5 to get 45. What was the number Lauulu started with?

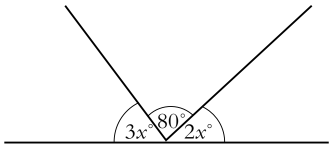
5.



   (a) Write down an expression for the area of the triangle.

   (b) What is x if the area is 15 m^2?

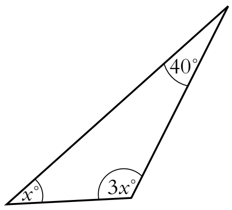
6. The diagram below shows three angles on a straight line:



(a) Write down an equation and use it to find x.

(b) Write down the sizes of the two unknown angles and check that the three angles shown add up to 180°.

7. Use an equation to find the sizes of the unknown angles in this triangle:



8. Peleseti thinks of a number, multiplies it by 3 and then adds 10. Her answer is 11 more than the number she thought of. If x is her original number, write down an equation and solve it to find x.

Unit 3: MEASUREMENT

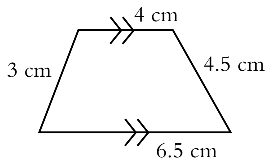
In this unit you will be:

Section 3.1 Calculating The Perimeter Of A Shape

The perimeter is the total distance around the edges of a shape.

Example 1

Calculate the perimeter of the trapezium.



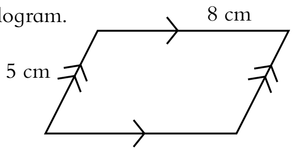
Solution

Perimeter $= 4 + 3 + 4.5 + 6.5$

$= 18$ cm

Example 2

Calculate the perimeter of the parallelogram.



Solution

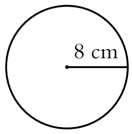
Perimeter $= 8 + 5 + 8 + 5$

$= 26$ cm

The perimeter of a circle is referred to as the 'circumference'. The circumference, C, of a circle $= 2\pi r$ or πd where r is the radius, d is the diameter of the circle, and $\pi = 3.14$.

Example 3

Calculate the circumference of a circle with radius 8 cm.



Solution

Using the formula, $C = 2\ \pi r$, gives

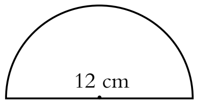
$C = 2 \times 3.14 \times 8$

$= 50.24$ cm

Example 4

The diagram shows a semicircle of diameter 12 cm.

Calculate the perimeter of the semicircle.



Solution

Length of curve $= 3.14 \times 12 \div 2$

$= 18.84$ cm

Straight edge $= 12$ cm

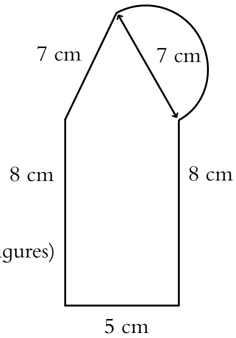
Total perimeter $= 12 + 18.84$

$= 30.84$ cm

$= 30.8$ cm (to 3 significant figures)

Example 5

The diagram shows a shape that is made up of a rectangle, a triangle and a semicircle. Calculate its perimeter.



Solution

Length of curve = 3.14   7   2

= 10.99 cm

Total perimeter = 8 + 5 + 8 + 7 + 10.99

= 38.99 cm

= 39.0 cm (to 3 significant figures)

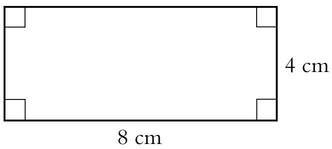
Skill Exercises: Perimeters

1. Giving your answer correct to 3 significant figures, calculate the circumference of a circle with:

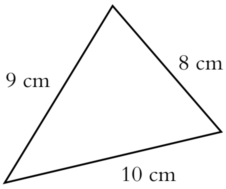
   (a) radius 6 m   (b) diameter 15 cm   (c) radius 8 mm

2. Calculate the perimeter of each of the following shapes:

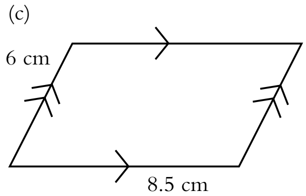
   (a)



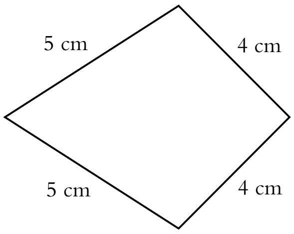
   (b)



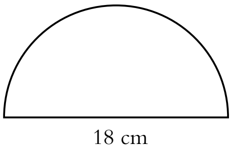
   (c)



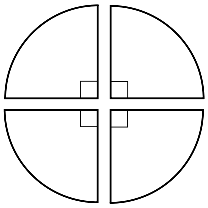
   (d)



3. Giving your answer correct to 3 significant figures, calculate the perimeter of the semicircle shown.



4. A circle of radius 8 cm is cut into four equal parts as shown in the diagram:

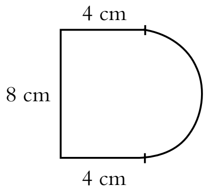


   (a) Calculate the circumference of the original circle, giving your answer correct to 2 decimal places.

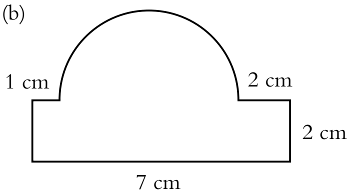
   (b) Calculate the perimeter of each of the 4 parts, giving your answers correct to 2 decimal places.

5. Calculate the perimeter of each of the following shapes, giving your answers correct to 1 decimal place. The circular parts are either semicircles or quarters of circles.

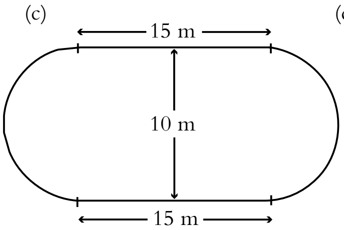
   (a)



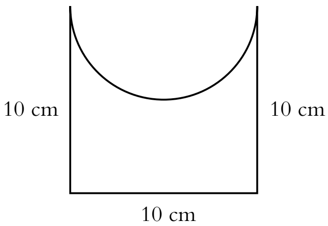
   (b)



   (c)

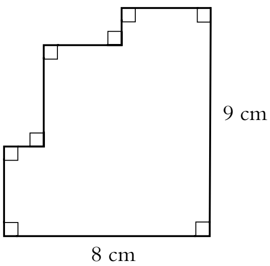


   (d)

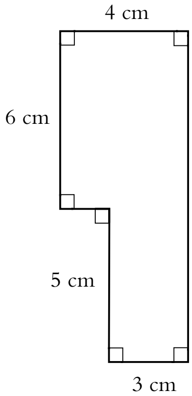


6. Calculate the perimeter of each of the following shapes:

   (a)

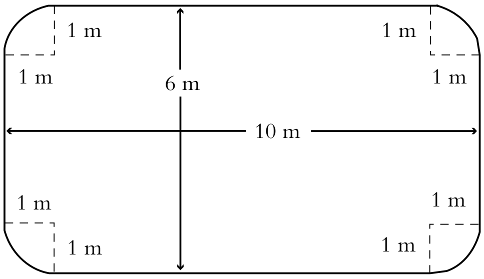


   (b)

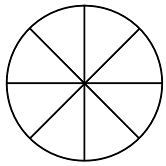


7. A square has an area of 36 m^2. Calculate its perimeter.

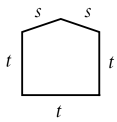
8. Calculate the perimeter of this shape, giving your answer correct to the nearest centimetre:



9. A circle of radius 32 cm is cut into 8 equal parts, as shown in the diagram.
Calculate the perimeter of each part, giving your answer correct to the nearest millimetre.



10.

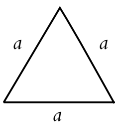


The perimeter of this shape is $3t + 2s$

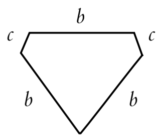
$$p = 3t + 2s$$

Write an expression for the perimeters of each of these shapes. Write each expression in its simplest form.

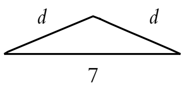
(a)



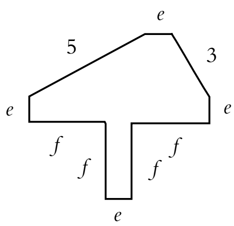
(b)



(c)

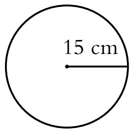


(d)



11. Tana and Sina are using their wheelchairs to measure distances.
   (a) The large wheel on Tana's wheelchair has a diameter of 60 cm.
   Tana pushes the wheel round exactly once.
   Calculate how far Tana has moved.
   Show your working.
   (b) The large wheel on Sina's wheelchair has a diameter of 52 cm.
   Sina moves her wheelchair forward 950 cm.
   Calculate how many times the large wheel goes round.
   Show your working.

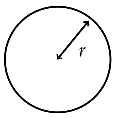
12. (a) A circle has a radius of 15 cm.
   Calculate the circumference of the circle.
   Show your working.



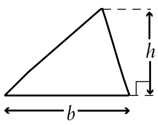
   (b) A different circle has a circumference of 120 cm.
   What is the radius of the circle?
   Show your working.

Section 3.2 Calculating The Area Of A Shape

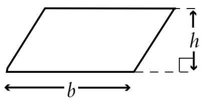
Area of a circle $= \pi r^2$



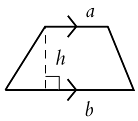
Area of a triangle $= \frac{1}{2}bh$
(h is perpendicular height)



Area of a parallelogram $= bh$
(h is perpendicular height)

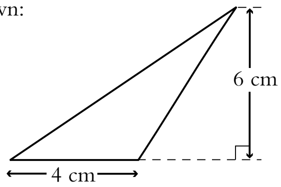


Area of a trapezium $= \frac{1}{2}(a + b)h$
(h is perpendicular height)



Example 1

Calculate the area of the triangle shown:



Solution

Area $= \frac{1}{2} \times 4 \times 6$

$= 12 \text{ cm}^2$

Example 2

Calculate the area of a circle with diameter 10 m.

Solution

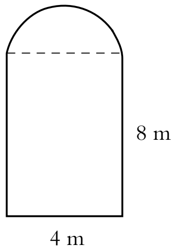
Radius $= 10 \div 2 = 5$ m

Area $= \pi \times 5^2 = 78.5 \text{ m}^2$

$= 78.5 \text{ m}^2$ (to 3 significant figures)

Example 3

Calculate the area of the shape shown:



Solution

Area of rectangle $= 4 \times 8$

$= 32 \text{ m}^2$

Radius of semicircle $= 4 \div 2 = 2$ m

Area of semicircle $= \frac{1}{2} \times \pi \times 2^2$

$= 6.28 \text{ m}^2$

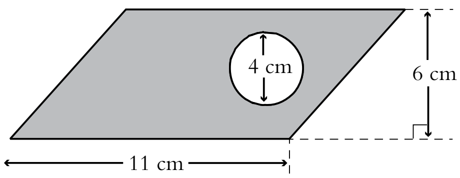
Total area $= 32 + 6.28 = 38.28 \text{ m}^2$

$= 38.3 \text{ m}^2$ (to 3 significant figures)

Example 4

The diagram shows a piece of card in the shape of a parallelogram, that has had a circular hole cut in it.

Calculate the area of the shaded part.



Solution

Area of parallelogram $= 11 \times 6$

$= 66 \text{ cm}^2$

Radius of circle $= 4 \div 2 = 2 \text{ cm}$

Area of circle $= \pi \times 2^2$

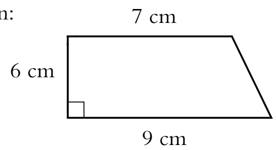
$= 12.56 \text{ cm}^2$

Area of shape $= 66 - 12.56 = 53.44 \text{ cm}^2$

$= 53.4 \text{ cm}^2$ (to 3 significant figures)

Example 5

Calculate the area of the trapezium shown:



There are two ways to find the area.

Solution 1: Use the formula

Area $= \frac{1}{2}(a + b)h$

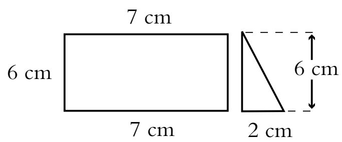
$= \frac{1}{2}(7 + 9) \times 6$

$= \frac{1}{2}(16) \times 6$

$= 48 \text{ cm}^2$

Solution 2: Find the area of each part of the shape and add them together.

This shape is made of a rectangle and a triangle.



Area of trapezium $=$ Area of rectangle $+$ Area of triangle

$= (6 \times 7) + \frac{1}{2}(2) \times 6$

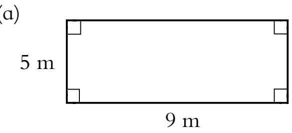
$= 42 + 6$

$= 48 \text{ cm}^2$

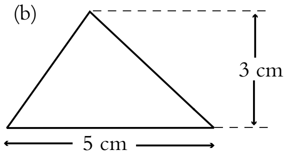
Skill Exercises: Areas

1. Calculate the area of each of the following shapes:

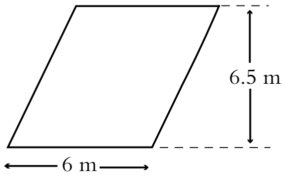
(a)



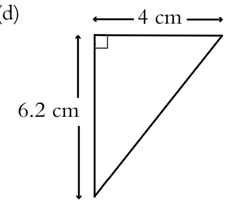
(b)



(c)



(d)

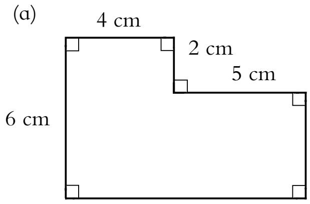


2. Calculate the area of a circle with:

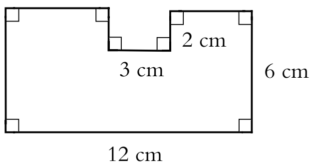
(a) radius 6 m (b) diameter 20 cm (c) diameter 9 cm

3. Calculate the area of each of the following shapes:

(a)



(b)



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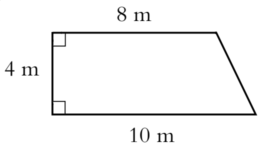
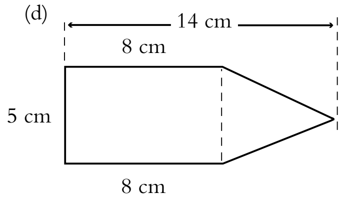


(d)

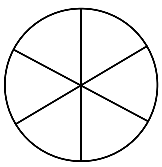


4. Calculate the area of a semicircle with:

(a) radius 30 cm (b) diameter 14 mm.

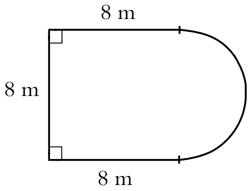
5. A circle of radius 8 cm is cut into 6 parts of equal size, as shown in the diagram.

Calculate the area of each part.

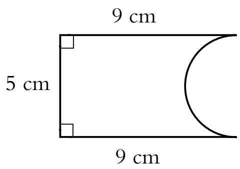


6. Calculate the area of each of the following shapes. Each of the curved parts is a semi-circle.

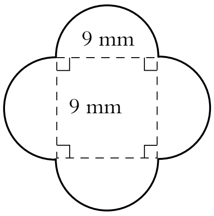
(a)



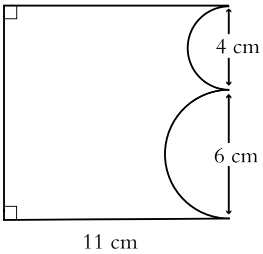
(b)



(c)

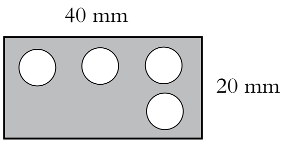


(d)

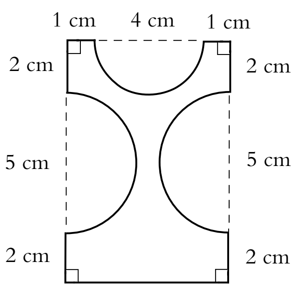


7. A rectangular metal plate is shown in the diagram. Four holes of diameter 8 mm are drilled in the plate.

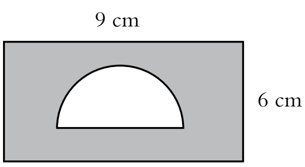
   Calculate the area of the remaining metal.



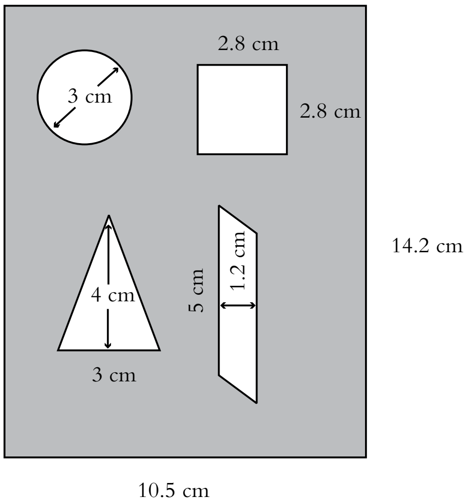
8. Calculate the area of the shape shown: Each of the curved parts is a semi-circle.



9. The area that has been shaded in the diagram has an area of 21.8 cm^2. Calculate the diameter of the semi-circular hole.

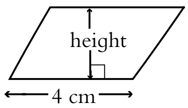


10. The diagram shows the lid of a box with some pieces cut out.
    Calculate the area of the lid.

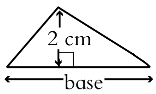


11. Each shape in this question has an area of 10 cm². No diagram is drawn to scale.

    (a) Calculate the height of the parallelogram.

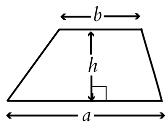


    (b) Calculate the length of the base of the triangle.



    (c) What might be the values of h, a and b in this trapezium?

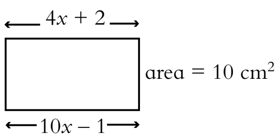
    What else might be the values of h, a and b?

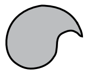


    (d) Look at this rectangle:

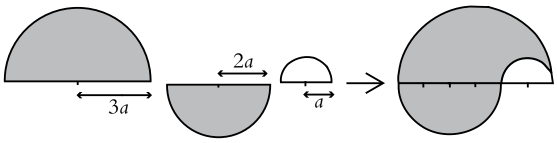
    Calculate the value of x and use it to find the length and width of the rectangle.

    Show your working.



12. This shape is designed using three semi-circles. 

The radii of the semi-circles are $3a$, $2a$ and a.



(a) Find the area of each semi-circle, in terms of a and π, and show that the total area of the shape is $6\pi a^2$.

(b) The area, $6\pi a^2$, of the shape is 12 cm^2.

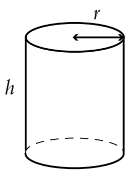
Write an equation in the form $a = \ldots$, leaving your answer in terms of π.

Show your working and simplify your equation.

Section 3.3 Calculating The Volume Of A Cylinder

Cylinder

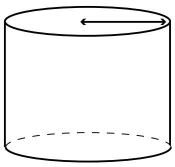
Volume $= \pi r^2 h$



Example

Calculate the volume of the cylinder shown:

$r = 4$ cm



$h = 6$ cm

$\pi \approx 3.14$

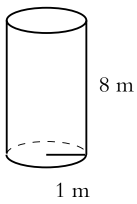
Solution

$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi \times 4^2 \times 6 \\ &= 96\,\pi \\ &= 301.44 \text{ cm}^3 \\ &= 301 \text{ cm}^3 \text{ (3 s.f.)} \end{aligned}$$

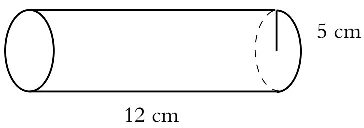
Skill Exercises: Volumes of Cylinders

1. Find the volumes of these cylinders. Take $\pi = 3.14$. Round the answers to 2dp.

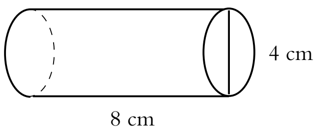
(a)



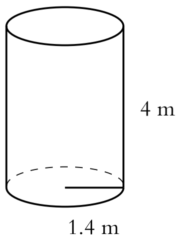
(b)



(c)

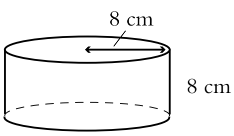


(d)

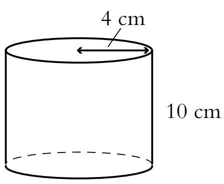


2. Calculate the volume of each of the following cylinders.

(a)

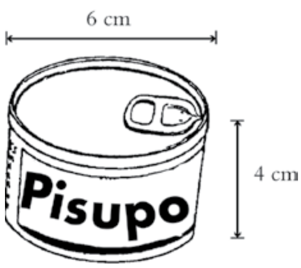


(b)



3. The internal measurements of a tin are shown.

Work out the volume of Pisupo food that the tin contains.
Show your working.



Unit 4: PROBABILITY AND STATISTICS

In this unit you will be:

4.1 Calculating simple probabilities

- Probabilities.
- Probability of a single event.

4.2 Calculating expected values

4.3 Estimating probabilities

Section 4.1 Calculating Simple Probabilities

Probabilities

Probabilities are used to describe how likely or unlikely it is that something will happen. For example, weather forecasters often talk about how likely it is to rain.

Example 1

(a) When you roll a dice, which number are you most likely to get?

(b) If you rolled a dice 600 times how many sixes would you expect to get?

(c) Would you expect to get the same number of ones?

Solution

(a) You are equally likely to get any of the six numbers.

(b) You would expect to get a six in about $\frac{1}{6}$ of the throws, so 100 sixes.

(c) Yes, in fact you would expect to get about 100 of each number.

Example 2

Use one of the following to describe each of the statements (a) to (d).

Certain

Very likely

Likely

Unlikely

Very unlikely

Impossible

(a) It will snow tomorrow.

(b) It will be sunny tomorrow.

(c) You win a car in a competition tomorrow.

(d) You are late for school tomorrow.

Solution

(a) Impossible. It has never snowed in Samoa.

(b) Likely, or Very likely in Samoa.

(c) Very unlikely if you have entered the competition. Impossible if you have not entered the competition.

(d) Very unlikely, unless the school bus breaks down.

Skill Exercises: Probabilities

1. If you toss a coin 500 times, how many times would you expect it to land:

   (a) on its side? (b) heads up? (c) tails up?

2. A tetrahedron is a shape with four faces. The faces are numbered 1, 2, 3 and 4. The tetrahedron is rolled 200 times. How many times would you expect it to land on a side numbered:

   (a) 4? (b) 2? (c) 5?

3. Describe each of the following events as:

   Impossible

   Unlikely

   Likely

   Certain

   (a) You roll a normal dice and score 7.

   (b) You fall off your bike on the way home from school.

   (c) You complete all your maths homework correctly.

   (d) Manu Samoa wins their next rugby match.

   (e) The school bus is on time tomorrow.

4. Describe two events that are:

   (a) Certain.

   (b) Impossible.

   (c) Likely to happen.

   (d) Unlikely to happen.

5. How many sixes would you expect to get if you rolled a dice:

   (a) 60 times?

   (b) 120 times?

   (c) 6000 times?

   (d) 3600 times?

6. Kolisi tossed a coin a large number of times and got 450 heads. How many times do you think he tossed the coin?

7. Perelini rolled a dice and got 250 twos.

   (a) How many times do you think she rolled the dice?

   (b) How many sixes do you think she got?

8. Semisi chooses a playing card from a pack of 52 cards 100 times. He replaces the card after each choice. How many times would you expect him to get:

   (a) a red card? (b) a black card?

   (c) a heart? (d) a diamond?

Probability of a Single Event

In this section we calculate the probabilities of single events. We consider cases where all the possible outcomes are equally likely. For example, when you roll a fair dice you are equally likely to get any of the six numbers. (The words 'fair' or 'unbiased' mean that all outcomes are equally likely.)

$$\text{Probability of an event} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

Example 1

When you roll a fair dice, what is the probability of getting:

(a) a five?

(b) an even number?

(c) a four or a five?

Solution

The possible outcomes when you roll a dice are the scores

1, 2, 3, 4, 5, 6

so there are six possible outcomes.

(a) In this case there is only one successful outcome, that is, a 5.

$$\text{Probability of a 5} \qquad P(5) = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$= \frac{1}{6}$$

(b) In this case there are three successful outcomes, 2, 4 or 6.

$$\text{Probability of an even number} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$P\ (\text{even}) = \frac{3}{6}$$

$$= \frac{1}{2}$$

(c) In this case there are two successful outcomes, 4 or 5.

$$\text{Probability of a 4 or a 5} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$P\ (4 \text{ or } 5) = \frac{2}{6}$$

$$= \frac{1}{3}$$

Example 2

A bag of sweets contains six mints and four éclairs. One sweet is taken at random from the bag. What is the probability that it is:

(a) a mint? (b) an éclair?

Solution

The total number of possible outcomes is ten as there are ten sweets in the bag.

(a) As there are six mints in the bag, there are six successful outcomes.

$$\text{Probability of mint} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$P\ (\text{mint}) = \frac{6}{10}$$

$$= \frac{3}{5}$$

(b) As there are four éclairs, there are four successful outcomes.

$$\text{Probability of an éclair} = \frac{\text{number of successful outcomes}}{\text{total number of outcomes}}$$

$$P\ (\text{éclair}) = \frac{4}{10}$$

$$= \frac{2}{5}$$

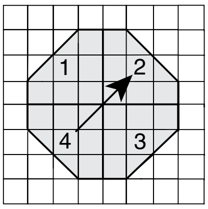
Skill Exercises: Probability of a Single Event

1. When you roll a fair dice, what is the probability that you obtain:
   (a) an odd number?
   (b) a 2?
   (c) a multiple of 3?
   (d) a number less than 5?
   (e) a number greater than 4?
   (f) a 3 or a number less than a 3?

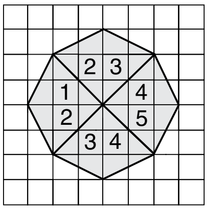
2. A bag contains six red balls and 14 blue balls. A ball is taken at random from the bag. What is the probability that it is:
   (a) a red ball? (b) a blue ball?

3. You toss a fair coin. What is the probability that you obtain a tail?

4. The diagram shows a spinner from a game. The black arrow spins and ends up pointing to one of the four numbers. What is the probability that it points to:
   (a) the number 1?
   (b) an even number?
   (c) a multiple of 3?



5. The diagram shows a spinner that is used in a board game. When the spinner is spun, what is the probability that it lands on:
   (a) 1?
   (b) 5?
   (c) 4?
   (d) an even number?
   (e) a number less than 4?



6. A bag of sweets contains eight mints, six toffees and two boiled sweets. A sweet is taken at random from the bag. What is the probability that it is:
   (a) a mint?
   (b) a toffee?
   (c) a boiled sweet?
   (d) not a mint?
   (e) not a toffee?

7. In a class there are 18 boys and 12 girls. One student is chosen at random to represent the class. What is the probability that this student is:

(a) a girl? (b) a boy?

8. The diagram shows a piece of card that is folded to form a dice. When the dice is rolled, what is the probability that it shows:

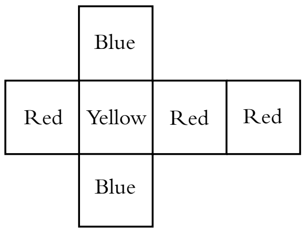
(a) a blue face?

(b) a red face?

(c) a yellow face?

(d) a face that is not red?

(e) a face that is not yellow?



9. The students in a class were asked to name their favourite colour. The results are given in the table:

Colour	**Number of Students**
Red	6
Black	2
Yellow	3
Green	4
Blue	10
Pink	7

If a student is picked at random from the class, what is the probability that their favourite colour is?

(a) red? (b) yellow? (c) pink?

(d) black? (e) not pink? (f) not green?

10. A bag contains six red balls and some white balls. When a ball is taken from the bag at random, the probability that it is red is $\frac{3}{5}$. How many white balls are in the bag?

Section 4.2 Calculating Expected Values

If we know the probability of an event we can estimate the number of times we expect that event to take place.

Expected value = probability of success total number of outcomes

Example 1

You toss an unbiased coin 500 times. How many heads should you expect to obtain?

Solution

Probability of a head P (head) $= \frac{1}{2}$

Expected number of heads $= \frac{1}{2} \quad 500$

$= 250$

Example 2

You roll a fair dice 120 times. How many times would you expect to obtain:

(a) a 6? (b) a multiple of 3?

Solution

(a) Probability of a 6 P (6) $= \frac{1}{6}$

Expected number of sixes $= \frac{1}{6} \quad 120$

$= 20$

(b) Probability of a multiple of 3 $= \frac{2}{6} = \frac{1}{3}$

Expected number of multiples of 3 $= \frac{1}{3} \quad 120$

$= 40$

Skill Exercises: Calculating Expected Values

1. If you roll an unbiased dice 600 times, how many times would you expect to obtain:

   (a) a one? (b) an even number?

   (c) an odd number? (d) a number less than three?

2. A spinner is marked with the numbers 1 to 5, each of which is equally likely to occur when the spinner is spun. If it is spun 200 times, how many times would you expect to obtain:

   (a) a five? (b) an even number?

   (c) a number less than three? (d) a prime number?

3. If the probability that it rains on a day in September is $\frac{1}{5}$, on how many days in September would you expect it to rain?

4. When you open a packet of sweets and take one out at random, the probability that it is blue is $\frac{1}{8}$. If you open 40 packets of sweets, how many times would you expect to take out a blue sweet first?

5. Some chip packets contain prizes. The probability that you find a prize in a chip packet is $\frac{1}{25}$. How many prizes would you expect to find if you opened:

   (a) 50 packets? (b) 200 packets? (c) 1000 packets?

6. The probability that Lomitusi misses the school bus is $\frac{3}{10}$. In a school year there are 40 weeks, each of five days.

   How many times can you expect Lomitusi to miss the bus in:

   (a) a 12-week term? (b) a school year?

7. The probability that a person, selected at random, has been trained in First Aid is $\frac{1}{50}$. How many people trained in First Aid would you expect to find in:

   (a) a crowd of 50 000 spectators at a football match?

   (b) an audience of 300 at a theatre?

   (c) a group of 50 onlookers at the scene of an accident?

8. The probability that a certain type of seed germinates is 0.7. How many seeds would you expect to germinate if you planted:

   (a) 20 seeds? (b) 70 seeds? (c) 1000 seeds?

9. The probability that Emma wins a game of 'Freecell' on her computer is $\frac{2}{5}$. She wants to be able to say that she has won five games. How many games should she expect to play before she wins five games?

10. Paulo says that the probability that he misses the school bus is $\frac{1}{10}$.

    (a) How many times would you expect him to miss the bus in four weeks?

    (b) In four weeks he actually misses the bus three times, which is not the same as your answer to (a). Explain why your answer to (a) is still correct.

Section 4.3 Estimating Probabilities

Some probabilities cannot be calculated as in the last section; for example, the probability that it will rain on 20 November cannot be found in this way. Probabilities can, however, be estimated using relative frequencies found from observations or from experiments.

$$\textit{Relative frequency} = \frac{\textit{number of successful trials}}{\textit{total number of trials}}$$

Example 1

Matiu decides to estimate the probability that toast lands butter-side-down when dropped. He drops a piece of buttered toast 50 times and observes that it lands butter-side-down 30 times.

Estimate the probability that the toast lands butter-side-down.

Solution

An estimate of the probability is given by the relative frequency. In this case it is

$$\frac{30}{50} = \frac{3}{5}$$

Example 2

Sara tosses a coin 200 times. She gets 108 heads and 92 tails. Using her results, estimate the possibility of obtaining:

(a) a head when the coin is tossed

(b) a tail when the coin is tossed

Solution

The relative frequency gives an estimate of the probability.

(a) Relative frequency $= \frac{108}{200} = \frac{27}{50}$

(b) Relative frequency $= \frac{92}{200} = \frac{23}{50}$

We would expect both these probabilities to be $\frac{1}{2}$, and here the estimates are close to that value, indicating that her coin may be a fair one.

Note: If you do more trials your estimated probability (relative frequency) will be more accurate.

Skill Exercises: Estimating Probabilities

1. Toss a coin 100 times. Total the number of heads and divide by 100.

   (a) Is your answer close to $\frac{1}{2}$?

   Put all the results for your class together and obtain a new estimate of the probability of obtaining a head.

   $$\frac{\text{total number of heads}}{\text{number of students} \quad 100 \text{ (throws)}}$$

   (b) Is your new estimate closer to $\frac{1}{2}$ than the estimate in (a)?

2. A drawing pin can land 'point up' or 'point down' when dropped. Carry out an experiment to find an estimate of the probability that a drawing pin lands 'point up'.

3. (a) Roll a dice 100 times and record the results you obtain.

   (b) Estimate the probability of obtaining each of the numbers on the faces of the dice.

   (c) Do you think that the probabilities that you obtain are reasonable?

   (d) Obtain more results by rolling the dice another 100 times. How do your probability estimates change as you use more results?

4. By considering the people in your class, estimate the probability that a person chosen at random is left-handed.

5. If it rained on 12 days in November last year, estimate the probability that it will rain on 20 November next year.

6. A calculator can be used to generate random digits. Lani generates 100 random digits with his calculator. He lists the results in the following table:

0	𝍸 II	5	𝍸 𝍸
1	𝍸 III	6	𝍸 IIII
2	𝍸 I	7	𝍸 𝍸 II
3	𝍸 𝍸 II	8	𝍸 𝍸
4	𝍸 𝍸 II	9	𝍸 𝍸 IIII

   Based on Lani's results, estimate the probability that the calculator produces:

   (a) 9

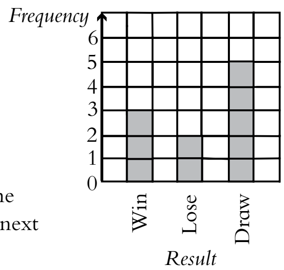
   (b) 2

   (c) a digit that is an odd number

   (d) a digit that is a prime number

7. Toni estimates the probability that there will be an empty space in the car park when he arrives at work is $\frac{4}{5}$. His estimate is based on 50 observations. On how many of these 50 days was he unable to find an empty space in the car park?

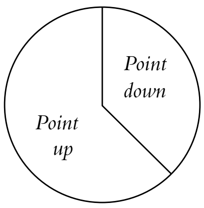
8. Petelo draws the bar chart opposite to show the results for his volleyball team so far this season.



(a) Use the bar chart to estimate the probability that his team will win their next match.

(b) Give reasons why this estimate of the probability that they will win their next match may not be very reliable.

9. Sasha carries out the drawing pin experiment described in question 2. She shows her results in this pie chart:



Use her results to estimate the probability that the pin lands 'point up'.

Unit 5: NUMBER – PART 2

In this unit you will be:

5.1 Using ideas of ratio and proportion

- Equivalent ratios.
- Direct proportion.
- Proportional division.
- Inverse proportion.

5.2 Applying the order of operations

5.3 Solving VAGST problems

Section 5.1 Using Ideas Of Ratio And Proportion

Equivalent Ratios

A ratio is usually represented by numbers separated by colons. For example, 4 : 5 is a ratio and is read as 'four to five'. Ratios are used when adding quantities together.

Orange concentrate is to be mixed with water in a ratio of 1 : 6. This means that for every unit of orange concentrate six units of water will be used. The table gives some examples:

Amount of Orange Concentrate (ml)	Amount of Water (ml)	Amount of Drink (ml)
1	6	**7**
20	120	**140**
5	30	**35**

The ratios 1 : 6 and 20 : 120 and 5 : 30 are all equivalent ratios, but 1 : 6 is the simplest form. This means $\frac{1}{7}$ of the total drink is orange concentrate and $\frac{6}{7}$ water.

Ratios can be simplified by dividing both sides by the same number. An alternative method for some purposes, is to reduce to the form 1 : n or n : 1 by dividing both numbers by either the left-hand-side (LHS) or the right-hand-side (RHS). For example:

the ratio 4 : 10 may be simplified to $\frac{4}{4} : \frac{10}{4} \Rightarrow 1 : 2.5$

the ratio 8 : 5 may be simplified to $\frac{8}{5} : \frac{5}{5} \Rightarrow 1.6 : 1$

Example 1

Write each of these ratios in its simplest form:

(a) 7 : 14 (b) 15 : 25 (c) 10 : 4

Solution

(a) Divide both sides by 7, giving:

$$7 : 14 = \frac{7}{7} : \frac{14}{7}$$

$$= 1 : 2$$

(b) Divide both sides by 5, giving:

$$15 : 25 = \frac{15}{5} : \frac{25}{5}$$

$$= 3 : 5$$

(c) Divide both sides by 2, giving:

$$10 : 4 = \frac{10}{2} : \frac{4}{2}$$

$$= 5 : 2$$

Example 2

Write these ratios in the form 1 : *n*:

(a) 3 : 12 (b) 5 : 6 (c) 10 : 42

Solution

(a) Divide both sides by 3, giving:

$$3 : 12 = 1 : 4$$

(b) Divide both sides by 5, giving:

$$5 : 6 = 1 : \frac{6}{5}$$

$$= 1 : 1.2$$

(c) Divide both sides by 10, giving:

$$10 : 42 = 1 : \frac{42}{10}$$

$$= 1 : 4.2$$

Example 3

The scale on a map is 1 : 20 000. What actual distance does a length of 8 cm on the map represent?

Solution

Actual distance = 8 20 000

= 160 000 cm

= 1600 m

= 1.6 km

Skill Exercises: Equivalent Ratios

1. Write each of these ratios in its simplest form:

(a) 2 : 6	(b) 4 : 20	(c) 3 : 15
(d) 6 : 2	(e) 24 : 4	(f) 30 : 25
(g) 14 : 21	(h) 15 : 60	(i) 20 : 100
(j) 80 : 100	(k) 18 : 24	(l) 22 : 77

2. Write in the form 1 : n, each of the following ratios:

(a) 2 : 5	(b) 5 : 3	(c) 10 : 35
(d) 2 : 17	(e) 4 : 10	(f) 8 : 20
(g) 6 : 9	(h) 15 : 12	(i) 5 : 12

3. Write in the form n : 1, each of the following ratios:

(a) 24 : 3	(b) 4 : 5	(c) 7 : 10
(d) 15 : 2	(e) 18 : 5	(f) 6 : 5

4. Iulia mixes 600 ml of orange juice with 900 ml of pineapple juice to make a fruit drink. Write down the ratio of orange juice to pineapple juice in its simplest form.

5. A builder mixes 10 shovels of cement with 25 shovels of sand. Write the ratio of cement to sand:

   (a) in its simplest form

   (b) in the form 1 : n

   (c) in the form n : 1

6. In a cake recipe, 300 grams of butter are mixed with 800 grams of flour. Write the ratio of butter to flour:

   (a) in its simplest form

   (b) in the form 1 : n

   (c) in the form n : 1

7. In a school there are 850 pupils and 40 teachers. Write the ratio of teacher to pupils:

   (a) in its simplest form

   (b) in the form 1: n

8. A map is drawn with a scale of 1 : 50 000. Calculate the actual distances, in km, that the following lengths on the map represent:

(a) 2 cm	(b) 9 cm	(c) 30 cm

9. A map has a scale of 1 : 200 000. The distance between two villages is 60 km. How far apart are the villages on the map?

10. On a map, a distance of 5 cm represents an actual distance of 15 km. Write the scale of the map in the form 1 : *n*.

Direct Proportion

Direct proportion can be used to carry out calculations like the one below:

If 10 calculators cost \$120

then 1 calculator costs \$12

and 8 calculators cost \$96

Example 1

If six copies of a book cost \$9, calculate the cost of eight books.

Solution

If 6 copies cost \$9

Then 1 copy costs $\frac{\$\,9}{6}$ = \$1.50

and 8 copies cost \$1.50   8 = \$12

Example 2

If 25 floppy disks cost \$5.50, calculate the cost of 11 floppy disks.

Solution

If 25 disks cost \$5.50 = 550s

then 1 disk costs $\frac{550\text{s}}{25}$ = 22s

so 11 disks cost 11   22s = 242s

= \$2.42

Skill Exercises: Direct Proportion

1. If five tickets for a play cost \$40, calculate the cost of:

   (a) 6 tickets   (b) 9 tickets   (c) 20 tickets

2. To make three glasses of orange drink you need 600 ml of water. How much water do you need to make:

   (a) 5 glasses of orange drink?

   (b) 7 glasses of orange drink?

3. If 10 litres of petrol cost \$8.20, calculate the cost of:

   (a) 4 litres (b) 12 litres (c) 30 litres

4. A baker uses 1800 grams of flour to make three loaves of bread. How much flour will he need to make:

   (a) 2 loaves? (b) 7 loaves? (c) 24 loaves?

5. Ben buys 21 pencils for 84 sene. Calculate the cost of:

   (a) 7 pencils (b) 12 pencils (c) 50 pencils

6. A 20 m length of rope costs \$14.40.

   (a) Calculate the cost of 12 m of rope.

   (b) What is the cost of the rope, per metre?

7. A window cleaner charges n cents to clean each window, and for a building with nine windows he charges \$4.95.

   (a) What is n?

   (b) Calculate the window cleaner's charge for a building with 13 windows.

8. Sixteen teams, each with the same number of people, enter a quiz. At the semifinal stage there are 12 people left in the competition.

   How many people entered the quiz?

9. Three identical buses can carry a total of 162 passengers. How many passengers in total can be carried on seven of these buses?

10. The total mass of 200 concrete blocks is 1460 kg. Calculate the mass of 900 concrete blocks.

Proportional Division

Sometimes we need to divide something in a given ratio. Mele and Sina share the profits from their business in the ratio 2 : 3. This means that, out of every \$5 profit, Mele get \$2 and Sina gets \$3.

Example 1

Siaki and Iulia run a stall at the market and take a total of \$90. They share the money in the ratio 4 : 5. How much money does each receive?

Solution

As the ratio is 4 : 5, first add these numbers together to see by how many parts the \$90 is to be divided.

$4 + 5 = 9$, so 9 parts are needed.

Now divide the total by 9.

$\frac{90}{9}$ = 10, so each part is \$10

Siaki gets 4 parts at \$10, giving 4   \$10 = \$40

Iulia gets 5 parts at \$10, giving 5   \$10 = <u>\$50</u>

\$90

Example 2

Lautele, Ben and Ema are given \$52. They decide to divide the money in the ratio of their ages 10 : 9 : 7. How much does each receive?

Solution

10 + 9 + 7 = 26 so 26 parts are needed.

Now divide the total by 26.

$\frac{\$52}{26}$ = 2, so each part is \$2

Lautele gets 10 parts at \$2, giving 10   \$2 = \$20

Ben gets 9 parts at \$2, giving 9   \$2 = \$18

Ema gets 7 parts at \$2, giving 7   \$2 = <u>\$14</u>

\$52

Skill Exercises: Proportional Division

1. (a) Divide \$50 in the ratio 2 : 3
   (b) Divide \$100 in the ratio 1 : 4
   (c) Divide \$60 in the ratio 11 : 4
   (d) Divide 80 kg in the ratio 1 : 3
2. (a) Divide \$60 in the ratio 6 : 5 : 1
   (b) Divide \$108 in the ratio 3 : 4 : 5
   (c) Divide 30 kg in the ratio 1 : 2 : 3
   (d) Divide 75 litres in the ratio 12 : 8 : 5
3. Fofoga and Perelini get \$80 by selling vegetables at the market. They divide the money in the ratio 2 : 3. How much money do they each receive?
4. In a chemistry lab, acid and water are mixed in the ratio 1 : 5. A bottle contains 216 ml of the mixture. How much acid and how much water were needed to make this amount of the mixture?
5. Blue and yellow paints are mixed in the ratio 3 : 5 to produce green. How much of each of the two colours are needed to produce 40 ml of green paint?

6. Simone, Sala and Matiu are given a total of \$300. They share it in the ratio 10 : 11 : 9. How much does each receive?

7. In a fruit drink, pineapple juice, orange juice and apple juice are mixed in the ratio 7 : 5 : 4. How much of each type of juice is needed to make:

   (a) 80 ml of the drink? (b) 1 litre of the drink?

8. Blue, red and yellow paints are mixed to produce 200 ml of another colour. How much of each colour is needed if they are mixed in the ratio:

   (a) 1 : 1 : 2? (b) 3 : 3 : 2? (c) 9 : 4 : 3?

9. To start up a small business, it is necessary to spend \$800. Paulo, Makareta and Tenisi agree to contribute in the ratio 8 : 1 : 7. How much does each need to spend?

10. Ana, Keleti and Aukuso share out 10 biscuits so that Ana has two, Keleti has six and Aukuso has the remainder. Later they share out 25 biscuits in the same ratio. How many does each have this time?

Inverse Proportion

Inverse proportion is when an increase in one quantity causes a decrease in another.

The relationship between speed and time is an example of inverse proportionality: as the speed increases, the journey time decreases, so the time for a journey can be found by dividing the distance by the speed.

Example 1

(a) Ben rides his bike at a speed of 10 km/h. How long does it take him to cycle 40 kilometres?

(b) On another day he cycles the same route at a speed of 16 km/h. How much time does the journey take?

Solution

(a) Time $= \dfrac{40 \text{ km}}{10 \text{ km/h}}$

$= 4$ hours

(b) Time $= \dfrac{40 \text{ km}}{16 \text{ km/h}}$

$= 2\frac{1}{2}$ hours

Note: *Faster speed* $\Rightarrow$ *shorter time.*

Example 2

Tai has to travel 280 kilometres. How long does it take if he travels at:

(a) 50 km/h?

(b) 60 km/h?

(c) How much time does he save when he travels at the faster speed?

Solution

(a) Time $= \dfrac{280 \text{ km}}{50 \text{ km/h}}$

$= 5.6$ hours

$= 5$ hours 36 minutes

(b) Time $= \dfrac{280 \text{ km}}{60 \text{ km/h}}$

$= 4\frac{2}{3}$ hours

$= 4$ hours 40 minutes

(c) Time saved $= 5$ hours 36 mins – 4 hours 40 mins

$= 56$ minutes

Example 3

In a factory, each employee can make 40 chicken pies in one hour. How long will it take:

(a) 6 people to make 40 pies?

(b) 3 people to make 240 pies?

(c) 10 people to make 600 pies?

Solution

(a) 1 person makes 40 pies in 1 hour

6 people make 40 pies in $\frac{1}{6}$ hour (or 10 minutes)

(b) 1 person makes 40 pies in 1 hour

1 person makes 240 pies in $\dfrac{240}{40} = 6$ hours

3 people make 240 pies in $\frac{6}{3} = 2$ hours

(c) 1 person makes 40 pies in 1 hour

1 person makes 600 pies in $\dfrac{600}{40} = 15$ hours

10 people make 600 pies in $\dfrac{15}{10} = 1\frac{1}{2}$ hours

Skill Exercises: Inverse Proportion

1. How long does it take to complete a journey of 300 kilometres travelling at:

   (a) 60 km/h? (b) 50 km/h? (c) 40 km /h?

2. Aleki has to travel 420 km. How much time does he save if he travels at 70 km/h rather than 50 km/h?

3. Sara has to travel 60 km to see her cousin. Her dad drives at 30 km/h and her uncle drives at 40 km/h. How much time does she save if she travels with her uncle rather than with her dad?

4. Tasi usually walks to school at 3 km/h. When Jennifer walks with him he walks at 4 km/h. He walks 1 km to school. How much quicker is his journey when he walks with Jennifer?

5. One person can put 200 letters into envelopes in one hour. How long would it take for 200 letters to be put into envelopes by:

   (a) 4 people? (b) 6 people? (c) 10 people?

6. A person can make 20 badges in one hour using a machine. How long would it take:

   (a) 4 people with machines to make 20 badges?

   (b) 10 people with machines to make 300 badges?

   (c) 12 people with machines to make 400 badges?

7. An aeroplane normally completes a 2700 km flight in $4\frac{1}{2}$ hours. How much faster would it have to fly to complete the journey in four hours?

8. On Monday Lomi takes 15 minutes to run two kilometres to school. On Tuesday he takes 20 minutes to run the same distance. Calculate his speed in km/h for each day's run.

9. Joshua shares a 2 kg tin of biscuits between himself and three friends.

   (a) How many kg of biscuits do they each receive?

   (b) How much less would they each have received if there were four friends instead of three?

10. Nadina and her friends can each make 15 Christmas cards in one hour. How long would it take Nadina and four friends to make:

    (a) 300 cards? (b) 1000 cards?

Section 5.2 Applying The Order Of Operations

In mathematics there is a definite order in which operations must be done.

For example, always do multiplication before addition.

To remember the order of operations use the word

BEDMAS

It gives the order in which operations should be completed.

1. B Work out anything in **B**rackets
2. E Work out numbers with **E**xponents
3. D Work out any **D**ivision or **M**ultiplication
   M in the order they occur. (From left to right)
4. A Finally, work out any **A**ddition or
   S **S**ubtraction in the order they occur

Example 1

Calculate:

(a) 3.5 + 2.5   4 (b) 4.3 + (6.5 – 3.7)

(c) 13.1 – 2.2   5 + 4.3 (d) 3 6.5^2

(e) (3.5 – 2.0)   4.2   7.0

Solution

(a) 3.5 + 2.5   4 = 3.5 + 10
 = 13.5

(b) 4.3 + (6.5 – 3.7) = 4.3 + 2.8
 = 7.1

(c) 13.1 – 2.2   5 + 4.3 = 13.1 – 11 + 4.3
 = 2.1 + 4.3
 = 6.4

(d) 3 6.5^2 = 3   42.25
 = 126.75

(e) (3.5 – 2.0)   4.2   7.0 = 1.5   4.2   7.0
 = 6.3   7
 = 0.9

Skill Exercises: Applying the Order of Operations

1. Calculate the following:

   (a) 3.7 + 7.8   5.1   (b) 70.5   14.1 + 2.9

   (c) 14.3 – 5.1   2.4 + 6.3   (d) 8.5   2.0   3.4

   (e) 3.1 – 2.2   (6.6 – 6.6)   (f) (7.4 – 2.3)   10.0   2.5

2. Calculate the following:

   (a) 1.2 10^{-1} + 2.3   (b) 4 $(^-4.1)^2 + 6.2$

   (c) $\sqrt{3.3^2}$ 1.2   (d) 10.0   5.5 – 10.0   3.5

Section 5.3 Solving VAGST Problems

This tax is added to the cost of many things you buy. In most shops the price marked includes this tax so you do not have to calculate it.

Occasionally, however, the prices are given without VAGST and it has to be added to the bill.

The present rate of this tax is 10% so to find the amount added on for VAGST, multiply the original price by 0.1.

The final price is 110% of the original price. To find the final price, multiply the original price by 1.1.

Example

1. A builder says he will charge $80 for doing a small job. To this, VAGST at 10% is added.

   What is the total cost?

   The VAGST is 10% of $80

   This is 0.1   80 = $8

   The total cost is $80 + $8 = $88

   Alternatively, you can find the total cost by multiplying the original price by 1.1.

   The original price is $80

   The total cost is 1.1   80 = $88

   The rate of tax might be changed. If it has, work out this example using the up-to-date rate.

Skill Exercises: Solving VAGST Problems

1. Find the VAGST at 10%, which must be added to these costs, and then find the total cost of each item.
   (a) A microwave oven costs $360 + VAGST
   (b) A plumber charges $32 + VAGST
   (c) Goods at a wholesale warehouse cost $124 + VAGST
   (d) The phone bill is $78 + VAGST
   (e) A builder charges $890 + VAGST

2. Perenise buys some tools marked $24. VAGST at 10% is added to this price.

   What is the total cost, including the tax?

3. Copy and complete this phone bill.

Call charges	$52.16
Line rental	$20.24
Subtotal excluding VAGST	
VAGST at 10%	______
Total amount now due	______

Unit 1: ANSWERS — NUMBER — PART 1

Section 1.1 Using Inequality Signs

(Pg.6) Skill Exercises: Inequality signs

1. (a) $5 < 8$ (b) $15 > 10$
   (c) $7 + 3 = 4 + 6$ (d) $3 + 4 > 5 + 1$
2. (a) $^-7 < {}^-2$ (b) $3 - 2 > {}^-5$
   (c) $3 - 5 > {}^-4 - 6$ (d) $0 > {}^-3$
3. (a) $x \in \{0, 1, 2, 3, 4, 5\}$ (b) $x \in \{8, 9, 10\}$
   (c) $x \in \{0, 1, 2, 3, 4\}$ (d) $x = 10$

Section 1.2 Applying The Laws Of Exponents

(Pg.7) Skill Exercises: Exponents

1. (a) 8 (b) 100 (c) 9 (d) 1000
   (e) 81 (f) 27 (g) 16 (h) 81
   (i) 49
2. (a) $10 \times 10 \times 10 \times 10 \times 10 = 10^5$
   (b) $3 \times 3 \times 3 \times 3 = 3^4$
   (c) $7 \times 7 \times 7 \times 7 \times 7 = 7^5$
   (d) $8 \times 8 \times 8 \times 8 \times 8 = 8^5$
   (e) $5 \times 5 = 5^2$
   (f) $19 \times 19 \times 19 \times 19 = 19^4$
   (g) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^7$
   (h) $11 \times 11 \times 11 \times 11 \times 11 \times 11 = 11^6$

3. (a) $8 = 2^3$ (b) $81 = 3^4$ (c) $100 = 10^2$
   (d) $81 = 9^2$ (e) $125 = 5^3$ (f) $1\ 000\ 000 = 10^6$
   (g) $216 = 6^3$ (h) $625 = 5^4$

4. No, because $10^2 = 100$ and $2^{10} = 1024$.

5. Yes, because $3^4 = 81$ and $4^3 = 64$.

6. No, because $5^2 = 25$ and $2^5 = 32$.

7. (a) $49 = 7^2$ (b) $64 = 4^3$ (c) $64 = 2^6$
   (d) $64 = 8^2$ (e) $100\ 000 = 10^5$ (f) $243 = 3^5$

8 (a) 12 (b) 32 (c) 13
   (d) 36 (e) 8000 (f) 1032

9. (a) 625 (b) 1 (c) 27 (d) 1331

10.(a) 10^5 (b) 2^{10} (c) 3^2 (d) 2^3
   (e) 10^4 (f) 5^2

11.(a) $k = 3$, $m = 6$ (b) 16 384

(Pg.10) Skill Exercises: The Laws of Exponents

1. (a) $2^3 \quad 2^7 = 2^{10}$ (b) $3^6 \quad 3^5 = 3^{11}$ (c) $3^7 \quad 3^4 = 3^3$
   (d) $8^3 \quad 8^4 = 8^7$ (e) $(3^2)^5 = 3^{10}$ (f) $(2^3)^6 = 2^{18}$
   (g) $\dfrac{3^6}{3^2} = 3^4$ (h) $\dfrac{4^7}{4^2} = 4^5$

2. (a) $a^3 \quad a^2 = a^5$ (b) $b^7 \quad b^2 = b^5$ (c) $(b^2)^5 = b^{10}$
   (d) $b^6 \quad b^4 = b^{10}$ (e) $(z^3)^9 = z^{27}$ (f) $\dfrac{q^{16}}{q^7} = q^9$

3. $9^4 = (3^2)^4 = 3^{2 \quad 4} = 3^8$

4. (a) 2 (b) 1 (c) 0 (d) 1

5. (a) $3^6 \quad 3^{11} = 3^{17}$ (b) $4^6 \quad 4^5 = 4^{11}$ (c) $\dfrac{a^6}{a^2} = a^4$
   (d) $(z^3)^6 = z^{18}$ (e) $(a^{19})^5 = a^{95}$ (f) $p^{16} \quad p^9 = p^7$
   (g) $(p^5)^8 = p^{40}$ (h) $q^{13} \quad q^{12} = q$

6. (a) 3 (b) 2 (c) 31 (d) 24
   (e) 875 (f) 48

7. (a) $8^2 = 2^6$ (b) $81^3 = 9^6 = 3^{12}$ (c) $25^6 = 5^{12}$
   (d) $4^7 = 2^{14}$ (e) $125^4 = 5^{12}$ (f) $1000^6 = 10^{18}$
   (g) $81 = 3^4$ (h) $256 = 4^4 = 2^8$

8. (a) $8 \quad 4 = 2^3 \quad 2^2 = 2^5$

(b) $25 \quad 625 = 5^2 \quad 5^4 = 5^6$

(c) $\frac{243}{9} = \frac{3^5}{3^2} = 3^3$

(d) $\frac{128}{16} = \frac{2^7}{2^4} = 2^3$

9. (a) False, $3^2 \quad 2^2 = 6^2$

(b) False, $5^4 \quad 2^3$ cannot be simplified as a single power

(c) True

(d) False, $\frac{10^8}{5^6}$ cannot be simplified as a single power

10. (a) $(2^6 \quad 2^3)^4 = (2^9)^4 = 2^{36}$

(b) $\left[\frac{3^6}{3^2}\right]^5 = (3^4)^5 = 3^{20}$

(c) $\left[\frac{2^3 \quad 2^4}{2^7}\right]^4 = (2^0)^4 = 2^0$

(d) $\left[\frac{3^2 \quad 9}{3^3}\right]^4 = (3^1)^4 = 3^4$

(e) $\left[\frac{6^2 \quad 6^8}{6^3}\right]^4 = (6^7)^4 = 6^{28}$

(f) $\left[\frac{7^8}{7^2 \quad 7^3}\right]^5 = (7^3)^5 = 7^{15}$

(Pg.13) Skill Exercises: Negative Exponents

1. (a) $\frac{1}{4}$ (b) $\frac{1}{8}$ (c) $\frac{1}{1000}$

(d) $\frac{1}{49}$ (e) $\frac{1}{64}$ (f) $\frac{1}{36}$

2. (a) $\frac{1}{49} = \frac{1}{7^2} = 7^{-2}$

(b) $\frac{1}{100} = \frac{1}{10^2} = 10^{-2}$

(c) $\frac{1}{81} = \frac{1}{9^2} = 9^{-2}$

(d) $\frac{1}{16} = \frac{1}{2^4} = 2^{-4}$

(e) $\frac{1}{10\,000\,000} = \frac{1}{10^7} = 10^{-7}$

(f) $\frac{1}{1024} = \frac{1}{2^{10}} = 2^{-10}$

3. (a) $\frac{7}{12}$ (b) $\frac{2}{3}$ (c) $\frac{1}{10}$

(d) $\frac{9}{1000}$ (e) $\frac{3}{20}$ (f) $\frac{13}{42}$

4. (a) 4^1 (b) 5^4 (c) 7^{10}

(d) 3^{-8} (e) 6^6 (f) 8^{-5}

(g) 7^4 (h) 8^{18}

5. (a) $\frac{1}{9} = 3^{-2}$ (b) $\frac{1}{100} = 10^{-2}$ (c) $\frac{1}{125} = 5^{-3}$

   (d) $\frac{5}{5^4} = 5^{-3}$ (e) $\frac{6^2}{6^3} = 6^{-1}$ (f) $\frac{2^2}{2^{10}} = 2^{-8}$

6. (a) x^5 (b) x^{-2} (c) x^{-4}

   (d) x^{-24} (e) x^{-8} (f) x^{-24}

7. (a) $0.1 = 10^{-1}$ (b) $0.25 = 2^{-2}$ (c) $0.0001 = 10^{-4}$

   (d) $0.2 = 5^{-1}$ (e) $0.001 = 10^{-3}$ (f) $0.02 = 50^{-1}$

8. (a) $\frac{x^4}{x^2} = x^2$ (b) $x^6 \quad x^{-4} = x^2$ (c) $x^9 \quad x^{-7} = x^2$

   (d) $\frac{x^7}{x^9} = x^{-2}$ (e) $\frac{x^3}{x^{-1}} = x^4$ (f) $(x^3)^{-2} = x^{-6}$

9. (a) $\frac{1}{8} = 2^{-3}$ (b) $\frac{1}{25} = 5^{-2}$ (c) $\frac{1}{81} = 9^{-2}$

   (d) $\frac{1}{10\,000} = 10^{-4}$

(Pg.15) Skill Exercise: Fractional Exponents

1. (a) 7 (b) 8 (c) 4 (d) $\frac{1}{9}$

   (e) $\frac{1}{10}$ (f) $\frac{1}{5}$ (g) 3 (h) $\frac{1}{6}$

   (i) 12

2. (a) 2 (b) $\frac{1}{2}$ (c) 5 (d) $\frac{1}{4}$

   (e) 6 (f) $\frac{1}{100}$

3. (a) 2 (b) $\frac{1}{8}$ (c) 10 (d) $\frac{1}{3}$

   (e) 5 (f) $\frac{1}{10}$

4. (a) 4 (b) 3 (c) 5

5. (a) False, because $16^{\frac{1}{2}} = 4$ (b) True

   (c) False, because $9 = 81^{\frac{1}{2}}$

(Pg.17) Calculator Skills: Exponents

1. (a) 25 (b) 36 (c) 1 (d) 225
2. (a) 6 (b) 12 (c) 16 (d) 100
3. (a) 216 (b) 1000 (c) 0.25 (d) 0.001
4. (a) 11 (b) 16 (c) 0.1 (d) 0.25
5. (a) 4 (b) 6 (c) 3 (d) 2

Section 1.3 Writing In Standard Form

(Pg.19) Skill Exercises: Standard Form

1. (a) 6210 (b) 8000 (c) 420 (d) 0.003
   (e) 0.06 (f) 0.0032 (g) 0.006 (h) 0.92
   (i) 0.036
2. (a) 2 10^{2} (b) 8 10^{3} (c) 9 10^{6} (d) 6.2 10^{4}
   (e) 8.4 10^{5} (f) 1.2 10^{10} (g) 6.18 10^{10} (h) 3.24 10^{6}
3. (a) 30 000 (b) 36 000 (c) 8200 (d) 310
   (e) 16 000 (f) 172 000 (g) 68 300 (h) 1 250 000
   (i) 9170
4. (a) 4 10^{-4} (b) 8 10^{-3} (c) 1.42 10^{-1} (d) 3.2 10^{-3}
   (e) 1.99 10^{-3} (f) 6.2 10^{-8} (g) 9.7 10^{-6} (h) 2.1 10^{-13}
5. (a) 0.06 (b) 0.7 (c) 0.0018 (d) 0.004
   (e) 0.0062 (f) 0.000981 (g) 0.667 (h) 0.0000386
   (i) 0.000000927
6. (a) 8 10^{9} (b) 6 10^{11} (c) 4.8 10^{-4} (d) 2.1 10^{-11}
   (e) 1.22 10^{2} (f) 1.28 10^{-13}
7. (a) 3 10^{3} (b) 4 10^{7} (c) 3 10^{1} (d) 2 10^{10}
   (e) 4 10^{9} (f) 4 10^{4}

(Pg.20) Calculator Skills: Standard Form

1. (a) $3\ 600\ 000 = 3.6 \quad 10^6$ (b) $9600 = 9.6 \quad 10^3$
   (c) $590\ 000 = 5.9 \quad 10^5$ (d) $0.089 = 8.9 \quad 10^{-2}$
   (e) $0.0086 = 8.6 \quad 10^{-3}$ (f) $0.00057 = 5.7 \quad 10^{-4}$
2. (a) $7.14 \quad 10^{10}$ (b) $4.92 \quad 10^{11}$ (c) $1.62 \quad 10^{13}$
   (d) $2.05 \quad 10^{19}$ (e) $6.144 \quad 10^{-5}$ (f) $2.38328 \quad 10^{44}$
3. (a) Statement (i) is the true one because $4 \quad 10^3 = 4000$ and $4^3 = 64$.
   (b) $0.36 \quad 10^5$ (c) $25 \quad 10^{-4}$
   (d) (i) $6 \quad 10^0$ (ii) $3 \quad 10^4$

Unit 2: ANSWERS — ALGEBRA

Section 2.1 Applying The Laws Of Exponents In Algebra

(Pg.24) Skill Exercises: Exponents in Algebra

1. (a) a^5	(b) a^{10}	(c) x^9	(d) x^2
(e) y^3	(f) p^3	(g) q^3	(h) x^8
(i) b^3	(j) b^6	(k) c^3	(l) x^5
(m) y^2	(n) $x^0 = 1$	(o) x^8	(p) p^4
(q) x^3	(r) y^4	(s) $x^0 = 1$	(t) $x^1 = x$
(u) x^{12}	(v) x^8	(w) x^{15}	(x) x^{54}
2. (a) $p = 3$	(b) $q = 0$		
3. (a) $8x^7$	(b) $32x^7$	(c) $3x^6$	(d) $12x^9$
4. (a) $3x^2$	(b) $5a^2$	(c) $2x^3$	(d) $2p$
5. (a) $9x^6$	(b) $16a^8$	(c) $2p^6$	(d) $3q^{-2}$

Section 2.2 Simplifying Algebraic Expressions

(Pg.25) Skill Exercises: Substitution

1. (a) 8	(b) 4	(c) 10	(d) 9
(e) 8	(f) 6	(g) 18	(h) 20
(i) 24	(j) 18	(k) 18	(l) 5
2. (a) 4	(b) $^-1$	(c) $^-5$	(d) 3
(e) $^-12$	(f) 2	(g) 6	(h) 0
(i) 13	(j) $^-2$	(k) 20	(l) 3

3. (a) 24 (b) 8 (c) 18 (d) 8
   (e) 40 (f) 35 (g) 14 (h) 48
   (i) 6 (j) 91 (k) $^-4$ (l) $^-6$
4. 60
5. 6
6. (a) 32 (b) 25 (c) 5 (d) 24
   (e) 2 (f) 18 (g) 27 (h) 0
   (i) 2.5 (j) 4
7. (a) $x = 23$ (b) $x = 34$ (c) $x = 50$ (d) $x = 0$
   (e) $x = 5$

(Pg.26) Problem Solving Skills: Substitution

1. 210
2. 2.5
3. 880
4. 150
5. 7

(Pg.28) Skill Exercises: Collecting Like Terms

1. (a) $5a$ (b) $13b$ (c) $2c$
   (d) $16d$ (e) $10e$ (f) f
   (g) 0 (h) $5p + 2h$ (cannot be simplified)
   (i) $a + 4b$ (j) $4x + 2y$ (k) $2t + 5s$
   (l) $3m + n + 9p + 11q$
2. (a) $a + b + c$ (b) $2a + b$ (c) $a + 2b + c$
   (d) $6a$ (e) $5b$ (f) $4a + 4b$
3. (a) $6a - 10$ (b) $18b + 15c$ (c) $4d^2 + 4d$
   (d) $5e^3 - 5e^2 + 10e$ (e) $3f^2 - 54f + 12$
4. (a) $3a + 3b$ (b) $c - 10d$ (c) $8e - f$
   (d) $^-3g - 9h$ (e) $j + 6k$ (f) $8p - 13q + r$
   (g) $44 - 5s$ (h) $x^2 - x - 6$ (i) $2x^2 - 9x + 4$
   (j) $x - x^2$

(Pg.29) Problem Solving Skills: Writing Formulae

1. $2x + 5$
2. Cost = $50 + 2x$
3. Cost = $1 + 0.5x$

Section 2.3 Solving Linear Equations

(Pg.31) Skill Exercises: Solving Linear Equations (x on one side)

1. (a) $x = 6$ (b) $x = 6$ (c) $x = 8$ (d) $x = 7$
   (e) $x = 9$ (f) $x = 8$ (g) $x = 24$ (h) $x = 45$
   (i) $x = 9$ (j) $x = {}^-2$ (k) $x = {}^-2$ (l) $x = 18$
   (m) $x = {}^-14$ (n) $x = 0$ (o) $x = 2$
2. (a) $x = 5$ (b) $x = 6$ (c) $x = 5$ (d) $x = 5$
   (e) $x = 7$ (f) $x = 3$ (g) $x = 7$ (h) $x = 9$
   (i) $x = 4$ (j) $x = {}^-2$ (k) $x = {}^-2$ (l) $x = {}^-6$
3. (a) $x = 1\frac{1}{3}$ (or $\frac{4}{3}$) (b) $x = 1\frac{2}{5}$ (or $\frac{7}{5}$) (c) $x = 2\frac{1}{2}$ (or $\frac{5}{2}$)
   (d) $x = \frac{3}{8}$ (e) $x = 1\frac{1}{2}$ (or $\frac{3}{2}$) (f) $x = 4\frac{1}{4}$ (or $\frac{17}{4}$)

(Pg.33) Solving Linear Equations (x on both sides)

1. (a) $x = 3$ (b) $x = 3$ (c) $x = 6$ (d) $x = 3$
   (e) $x = 9$ (f) $x = 6$ (g) $x = 7$ (h) $x = 11$
   (i) $x = {}^-5$ (j) $x = {}^-3$ (k) $x = 10$ (l) $x = {}^-1$

(Pg.34) Solving Linear Equations (with brackets)

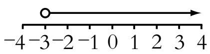
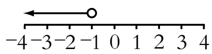
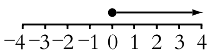
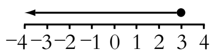
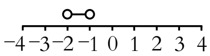
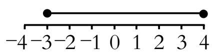
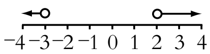
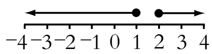
1. (a) $x = 1$ (b) $x = 16$ (c) $x = {}^-1$
   (d) $x = 2$ (e) $x = 2.5$ or $\frac{5}{2}$ (f) $x = 7\frac{2}{3}$ or $\frac{23}{3}$
   (g) $x = 6.4$ or $\frac{32}{5}$ (h) $x = 1.2$ or $\frac{6}{5}$
2. (a) $x = 4.3$ or $\frac{43}{10}$ (b) $x = 1$ (c) $x = 4.5$ or $\frac{9}{2}$
   (d) $x = 7.5$ or $\frac{15}{2}$
3. (a) $x = 2$ (b) $x = 4$ (c) x = 1
   (d) $x = 1$ (e) $x = 0.5$ or $\frac{1}{2}$ (f) $x = 1\frac{2}{3}$ or $\frac{5}{3}$
4. (a) $x =5$ (b) $x = \frac{3}{2}$ or $1\frac{1}{2}$ (c) $x = 1.2$ or $\frac{6}{5}$
   (d) $x = 2$

Section 2.4 Solving Linear Inequalities

(Pg.35) Skill Exercises: Writing Inequalities

1. (a) x is greater than 7 (b) x is less than or equal to 8
   (c) x is less than 1 (d) x is greater than 1 and less than 4
   (e) x is greater than or equal to $^-5$
2. (a) $x < 6$ (b) $x \geq {^-2}$ (c) $x > 0$
   (d) $^-3 < x < 10$ (e) $x \leq 5$
3. (a) 4, 5, 6 (b) 4, 5 (c) $^-2, ^-1, 0, 1, 2$
   (d) $^-7, ^-6, ^-5$ (e) 0, 1, 2, 3, 4, 5 (f) 2, 3, 4

(Pg.36) Skill Exercises: Showing Inequalities on a Number Line

1. (a)  (b) 
   (c)  (d) 
   (e)  (f) 
   (g)  (h) 
2. (a) $^-2, ^-1, 0, 1, 2, 3, 4$ (b) $^-4, ^-3, ^-2$
   (c) 0, 1, 2, 3, 4 (d) $^-4, ^-3, ^-2, ^-1, 0, 1, 2, 3$
   (e) no values (f) $^-3, ^-2, ^-1, 0, 1, 2, 3, 4$
   (g) $^-4, 3, 4$ (h) $^-4, ^-3, ^-2, ^-1, 0, 1, 2, 3, 4$

(Pg.37) Skill Exercises: Solving Linear Inequalities

(a) $x < 8$ (b) $x < {^-6}$ (c) $x \geq \frac{1}{2}$ (d) $x \leq {^-4}$
(e) $x > 6$ (f) $x \leq \frac{3}{4}$ (g) $x > 11\frac{1}{2}$ (h) $x \geq {^-1}$
(i) $x > 1$ (j) $x \leq 5$

Section 2.5 Writing Linear Equations

(Pg.38) Skill Exercises: Writing Linear Equations

1. $3(x + 4) = 18 \Rightarrow x = 2$

2, (a) $5(x + 7) = 55$ (b) $x = 4$

3. (a) $4(x + 6) = 17$ (b) $x = {}^{-}1.75$ or $\frac{^{-}7}{4}$

4. $5(11 - x) = 45 \Rightarrow x = 2$

5. (a) Area $= \frac{1}{2} \times 3 \times (x + 4) = \frac{3}{2}(x + 4)$

   (b) $x = 6$

6. (a) $$\begin{aligned} 3x + 80 + 2x &= 180 \\ 5x + 80 &= 180 \end{aligned}$$

   (b) $x = 20$, so angles are 60°, 80°, 40°

7. $$\begin{aligned} 4x + 40 &= 180 \\ 4x &= 140 \\ x &= 35^\circ \end{aligned}$$, so angles are 35°, 105°, 40°

8. $$\begin{aligned} 3x + 10 &= x + 11 \\ 2x &= 1 \\ x &= \tfrac{1}{2} \end{aligned}$$

Unit 3: ANSWERS — MEASUREMENT

Section 3.1 Calculating The Perimeter Of A Shape

(Pg.42) Skill Exercises: Perimeters

1. (a) 37.7 m (b) 47.1 cm (c) 50.2 mm
2. (a) 24 cm (b) 27 cm (c) 29 cm (d) 18 cm
3. 46.3 cm
4. (a) 50.24 cm (b) 28.56 cm
5. (a) 28.6 cm (b) 20.3 cm (c) 61.4 m (d) 45.7 cm
6. (a) 34 cm (b) 30 cm
7. 24 m
8. 3028 cm
9. 89.1 cm
10. (a) $p = 3a$ (b) $p = 3b + 2c$ (c) $p = 2d + 7$ (d) $p = 4e + 4f + 8$
11. (a) Distance moved = wheel circumference = $\pi \times 60$
    = 188.4 cm
    (b) Wheel circumference = $\pi \times 52 = 163.28$ cm
    Number of turns = $950 \div 163.28 = 5.818226$
    = 5.82 turns (to 3 s.f.)
12. (a) Circumference = $2\pi \times 15$
    = 94.2 cm (to 3 s.f.)
    (b) Radius = $120 \div (2 \times \pi)$
    = 19.1 cm (to 3 s.f.)

Section 3.2 Calculating The Area Of A Shape

(Pg.48) Skill Exercises: Areas

1. (a) 45 m^2 (b) 7.5 cm^2 (c) 39 m^2 (d) 12.4 cm^2
2. (a) 113 m^2 (b) 314 cm^2 (c) 63.6 cm^2
3. (a) 44 cm^2 (b) 66 cm^2 (c) 36 m^2 (d) 55 cm^2
4. (a) 1413 cm^2 (b) 76.9 mm^2
5. 33.49 cm^2
6. (a) 89.1 m^2 (b) 35.19 cm^2 (c) 208.17 mm^2 (d) 89.6 cm^2
7. 599.04 mm^2
8. 28.1 cm^2
9. 9.1 cm
10. 122.2 cm^2
11. (a) 2.5 cm (b) 10 cm

    (c) Any set of values for which $a > b$ and $(a + b) \times h = 20$, e.g. $h = 2$, $a = 6$, $b = 4$

    (d) $4x + 2 = 10x - 1 \Rightarrow 4x + 3 = 10x \Rightarrow 3 = 6x \Rightarrow x = 0.5$

    Length $= 4x + 2 = 4 \times 0.5 + 2 = 4$ cm

    Width $= \dfrac{\text{area}}{\text{length}} = \dfrac{10}{4} = 2.5$ cm

12. (a) Area first semicircle $= \pi \times (3a)^2 \div 2 = 4.5\pi a^2$

    Area second semicircle $= \pi \times (2a)^2 \div 2 = 2\pi a^2$

    Area third semicircle $= \pi \times (a)^2 \div 2 = 0.5\pi a^2$

    Total area $=$ first semicircle + second semicircle – third semicircle

    $= 4.5\pi a^2 + 2\pi a^2 - 0.5\pi a^2 = 6\pi a^2$

    (b)
$$6\pi a^2 = 12$$
$$a^2 = \frac{12}{6\pi}$$
$$a^2 = \frac{2}{\pi}$$
$$a = \sqrt{\frac{2}{\pi}}$$

Section 3.3 Calculating The Volume Of A Cylinder

(Pg.52) Skill Exercises: Volumes of Cylinders

1. (a) 25.12 m^3 (b) 942 cm^3 (c) 100.48 cm^3 (d) 24.62 m^3
2. (a) 1607.68 cm^3 (b) 502.4 cm^3
3. 113.04 cm^3

Unit 4: ANSWERS — PROBABILITY AND STATISTICS

Section 4.1 Calculating Simple Probabilities

(Pg.55) Skill Exercises: Probabilities

1. (a) 0 (b) about 250 (c) about 250
2. (a) 50 (b) 50 (c) 0
3. (a) Impossible (b) Unlikely
   (c) Likely or Unlikely (d) Likely or Unlikely
   (e) Likely
5. (a) 10 (b) 20 (c) 1000 (d) 600
6. About 900
7. (a) about 1500 (b) about 250
8. (a) 50 (b) 50 (c) 25 (d) 25

(Pg.58) Skill Exercises: Probability of a Single Event

1. (a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{3}$ (d) $\frac{2}{3}$
   (e) $\frac{1}{3}$ (f) $\frac{1}{2}$
2. (a) $\frac{3}{10}$ (b) $\frac{7}{10}$
3. $\frac{1}{2}$
4. (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$

5. (a) $\frac{1}{8}$ (b) $\frac{1}{8}$ (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

   (e) $\frac{5}{8}$

6. (a) $\frac{1}{2}$ (b) $\frac{3}{8}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

   (e) $\frac{5}{8}$

7. (a) $\frac{2}{5}$ (b) $\frac{3}{5}$

8. (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{1}{2}$

   (e) $\frac{5}{6}$

9. (a) $\frac{3}{16}$ (b) $\frac{3}{32}$ (c) $\frac{7}{32}$ (d) $\frac{1}{16}$

   (e) $\frac{25}{32}$ (f) $\frac{7}{8}$

10. 4

Section 4.2 Calculating Expected Values

(Pg.60) Skill Exercises: Calculating Expected Values

1. (a) 100 (b) 300 (c) 300 (d) 200
2. (a) 40 (b) 80 (c) 80 (d) 120
3. 6
4. 5
5. (a) 2 (b) 8 (c) 40
6. (a) 18 (b) 60
7. (a) 1000 (b) 6 (c) 1
8. (a) 14 (b) 49 (c) 700
9. 13
10. (a) 2, assuming he goes to school five days a week.

    (b) Because the expected number of times missed is a long term average; sometimes he might miss the bus three times, as here, and other times he might miss it once, twice or not at all.

Section 4.3 Estimating Probabilities

(Pg.63) Skill Exercises: Estimating Probabilities

1. (a) Your answer should be close to $\frac{1}{2}$

   (b) Your new answer should be closer to $\frac{1}{2}$

2. You will have your own result.

3. (d) Should get closer to $\frac{1}{6}$ if the dice is fair.

5. $\frac{2}{5}$

6. (a) $\frac{7}{50}$ (b) $\frac{3}{50}$ (c) $\frac{14}{25}$ (d) $\frac{2}{5}$

7. 10

8. (a) $\frac{3}{10}$

   (b) The estimate was based on only a small number of games. It also reflects the teams already played and, in the next match, they may play a stronger or weaker team than those they have played so far.

9. Approximately 0.65 or $\frac{2}{3}$.

Unit 5: ANSWERS — NUMBER – PART 2

Section 5.1 Using Ideas Of Ratio And Proportion

(Pg.68) Skill Exercises: Equivalent Ratios

1. (a) 1 : 3 (b) 1 : 5 (c) 1 : 5 (d) 3 : 1
   (e) 6 : 1 (f) 6 : 5 (g) 2 : 3 (h) 1 : 4
   (i) 1 : 5 (j) 4 : 5 (k) 3 : 4 (l) 2 : 7
2. (a) 1 : 2.5 (b) 1 : 0.6 (c) 1 : 3.5 (d) 1 : 8.5
   (e) 1 : 2.5 (f) 1 : 2.5 (g) 1 : 1.5 (h) 1 : 0.8
   (i) 1 : 2.4
3. (a) 8 : 1 (b) 0.8 : 1 (c) 0.7 : 1 (d) 7.5 : 1
   (e) 3.6 : 1 (f) 1.2 : 1
4. 600 : 900 = 2 : 3
5. (a) 2 : 5 (b) 1 : 2.5 (c) 0.4 : 1
6. (a) 3 : 8 (b) 1 : 2.67 or 1 : $2\frac{2}{3}$ (c) 0.375 : 1
7. (a) 4 : 85 (b) 1 : 21.25
8. (a) 1 km (b) 4.5 km (c) 15 km
9. 1 cm on map = 2 km 60 km = 30 cm on map
10. 1 cm on map = 3 km Scale = 1 : 300 000

(Pg.69) Skill Exercises: Direct Proportion

1. (a) \$48 (b) \$72 (c) \$160
2. (a) 1000 ml or 1 litre (b) 1400 ml or 1.4 litres
3. (a) \$3.28 (b) \$9.84 (c) \$24.60
4. (a) 1200 grams or 1.2 kg (b) 4200 grams or 4.2 kg
   (c) 14 400 grams or 14.4 kg

5. (a) 28s (b) 48s (c) $2.00
6. (a) $8.64 (b) 72s per m
7. (a) $n = 55$ (b) $7.15
8. 48 people
9. 378 people
10. 6570 kg

(Pg.71) Skill Exercises: Proportional Division

1. (a) $20 : $30 (b) $20 : $80 (c) $44 : $16
   (d) 20 kg : 60 kg
2. (a) $30 : $25 : $5 (b) $27 : $36 : $45 (c) 5 kg : 10 kg : 15 kg
   (d) 36 litres : 24 litres : 15 litres
3. $32 : $48
4. 36 ml : 180 ml (acid : water)
5. 15 ml : 25 ml (blue : yellow)
6. $100 : $110 : $90
7. (a) 35 ml : 25 ml : 20 ml (b) 437.5 ml : 312.5 ml : 250 ml
8. (a) 50 ml : 50 ml : 100 ml (b) 75 ml : 75 ml : 50 ml
   (c) 112.5 ml : 50 ml : 37.5 ml
9. $400 : $50 : $350
10. 5 : 15 : 5

(Pg.74) Skill Exercises: Inverse Proportion

1. (a) 5 hours (b) 6 hours (c) 7.5 hours
2. Time at 50 km/h = 8.4 hours (8 hours 24 mins)
   Time at 70 km/h = 6 hours (6 hours 0 mins)
   Time saved = 2 hours 24 mins
3. Time at 30 km/h = 2 hours (2 hours 0 mins)
   Time at 40 km/h = 1.5 hours (1 hour 30 mins)
   Time saved 30 mins
4. Time on own = 20 mins
   Time with Jennifer = 15 mins
   Time saved = 5 mins

5. (a) 15 mins (b) 10 mins (c) 6 mins
6. (a) 15 mins (b) $1\frac{1}{2}$ hours
   (c) $1\frac{2}{3}$ hours (1 hour 40 mins)
7. Normal speed = 600 km/h
   Increased speed = 675 km/h
   Speed increase = 75 km/h
8. Speed on Monday = 8 km/h Speed on Tuesday = 6 km/h
9. (a) 0.5 kg (b) 0.1 kg less (0.4 kg each)
10. (a) 4 hours (b) $13\frac{1}{3}$ hours = 13 hours 20 mins

Section 5.2 Applying The Order Of Operations

(Pg.76) Skill Exercises: Applying the Order of Operations

1. (a) 43.48 (b) 7.9 (c) 8.36 (d) 5.0
   (e) 3.1 (f) 20.4
2. (a) 2.42 (b) 73.44 (b) 3.96 (d) 20.0

Section 5.3 Solving VAGST Problems

(Pg.77) Skill Exercises: Solving VAGST Problems

1. (a) VAGST = $36.00 Total Cost = $396.00
   (b) VAGST = $3.20 Total Cost = $35.20
   (c) VAGST = $12.40 Total Cost = $136.40
   (d) VAGST = $7.80 Total Cost = $85.80
   (e) VAGST = $89.00 Total Cost = $979.00
2. Total Cost = $26.40
3. Sub Total = $72.40
   VAGST = <u>$7.24</u>
   Total Amount = $79.64